Gerasimovich
Vol.1 - Methods of
Astrophysical and
Astrophotographic
Investigations:-
Introduction
Theoretical physics
Ambartsumian

Refractors and Reflectors
Balanovsky

Astrophotography
Kostinsky

Chapter I:-

Foundations of
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COURSE IN ASTROPHYSICS AND STELLAR ASTRONOMY

by

B. P. Gerasomivich, Director of Pulkovo Observatory, Editor

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Volume I.

Methods of Astrophysical and Astrophotographic Investigations

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Preface

Astrophysics and stellar astronomy, which have undergone an extraordinary development during the last fifteen years, have changed considerably the science of cosmic phenomena, and have opened before us perspectives which recently it was difficult even to contemplate. Being closely connected with the latest advances in physics, they revealed to us cosmic laboratories where there exist temperatures and pressures which are outside the range of the usual conditions of physical experiments. Basing themselves on rich observational data, which was gathered during a long period, they permitted us to penetrate into the most distant spaces of the universe, and they transferred the problem of dynamics and of the structure of the stellar system from the realm of hypotheses into the field of real scientific investigations.

This course is the first attempt in world of scientific literature at a complete, connected and yet non-elementary exposition of the methods and results of astrophysics and stellar astronomy. The pioneering character of this course is its distinction, yet it is also a source of possible shortcomings. The authors were unable to take advantage of classical standard texts because they do not exist as yet. On the other hand, the quickly changing aspects of our science contain the dangers of anachronisms as well as too hasty conclusions. The authors tried to maintain the greatest objectivity possible with regard to debatable and hotly discussed problems.

The course is composed by a group of Pulkovo astronomers on the principle of specialization. We may assume, that this principle should increase the quality of the course, although it was sometimes difficult to synchronize the stype of the several authors and to escape some repetitions. The first part contains the exposition of the methods of astrophysics and astrophotography. The second part contains the physics of the solar system and the stellar astronomy.

B. Gerasimovich

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B. Gerasimovich

Introduction

1. Some Information from Theoretical Physics.

By V. A. Ambartsumian

Because of those conditions under which the substance of stars, nebulae and other heavenly bodies exists, its physical condition in many cases differs considerably from the ideal conditions of matter studied by thermodynamics.

The latter deals with equilibrium conditions of matter contained in a constant volume the walls of which are impenetrable to matter as well as to radiant energy. Such conditions are called the conditions of thermodynamic equilibrium. Only in the inner parts of the stars is matter so situated as to be close to the conditions of thermodynamic equilibrium. Because of the great opacity of stellar matter inside a star conditions are created similar to those which obtain in a volume with walls impenetrable to radiation. In Stellar atmospheres, especially in their outer layers - - the chromosphere, - in gaseous nebulae, and in interstellar space matter exists in conditions which greatly differ from those of thermodynamic equilibrium inasmuch as the flow of energy penetrating the substance and going out into space does not permit us to regard a given object as a closed system, not to speak of those objects which, in addition to not being in a condition of thermodynamic equilibrium, are also in a non-static condition in general.

On account of what we have said above, the usual thermodynamic methods of describing the condition of matter (presumption of knowledge of temperature and pressure) are, in general, not applicable to stellar atmospheres. The non-equilibrium conditions are of greater variety than the totatity of all possible equilibrium conditions depending on two parameters. However, even in these cases the conception of temperature is used. But in using it the meaning thereof must be indicated or implied (for instance, the quantity which is proportional to the mean kinetic energy of the particles; the parameter characterizing the composition of the radiation, etc). We shall give later some of the most important properties of matter in equilibrium conditions. We shall be interested in gases only. Although liquid and solid matter exists in universal space (planets, for instance) astrophysics has studied it but little as yet.

1. Laws of Distribution. Let us assume that our gas consists of identical particles which do not interact, and the condition of each at a given moment t may be characterized by s given coordinates (q) and by s impulses (p):

$$q_1, q_2, \cdots q_s, \quad p_1, \cdots\cdots p_s$$

or, geometrically speaking, let the condition of each particle be characterized by some point in space of 2s diamensions (phase space). The energy of a particle ε is a function of these quantities only:

$$\varepsilon = \varepsilon(q_1, q_2, \cdots q_s, p_1, \cdots p_s).$$

The microscopic condition of the entire gas may be given by indicating the position of the points representing the particles in the phase space. Each distribution of the points characterizes some condition of the gas. In theromodynamic equilibrium the most probable distribution takes place, i.e. that which may occur in the greatest number of ways. Let

$$f(q_1, q_2, \cdots q_s, p_1, \cdots p_s)\, dq_1 \ldots, dq_s\, dp_1 \ldots dp_s \tag{1}$$

be the number of particles, and let the point representing them inside the volume be dq_1, dq_2, ... dq_s dp_1, ...dp_s in the most probable distribution. This function must satisfy the two conditions:

$$\int f(q,p)\, dq\, dp = N \tag{2}$$

$$\int f(q,p)\, \varepsilon(q,p)\, dq\, dp = E\,, \tag{3}$$

where the integrals cover the entire phase space, N is the number of particles composing the gas, E is the total energy of the gas.

The given parameters N and E determine the condition of the gas in thermodynamic equilibrium. Since these quantities may take any values, the function f(q, p) must contain two constants to whose corresponding changes the conditions (2) and (3) may be made to conform.

The form of function f(q, p) depends upon whether the particles composing the gas have an even or odd charge if we take the electron charge as unit. If the particles have an even number charge then

$$f(q,p) = \frac{1}{Ce^{\frac{\varepsilon(q,p)}{kT}} - 1} \cdot \frac{1}{h^s}\,, \tag{4}$$

where k and h are universal constants, and C and T are parameters of distribution determined from the conditions (2) and (3). Parameter T is called the temperature of the gas. The gas under consideration consisting of particles having an even numbered charge is subject to Bose's statistics. In particular, a gas consisting of neutral atoms or molecules is also subject to Bose's statistics.

The following is the function for the gas consisting of particles having an odd-numbered charge

$$f(q,p) = \frac{1}{Ce^{\frac{\varepsilon(q,p)}{kT}}+1} \cdot \frac{1}{h^s}, \qquad (5)$$

and it is said that the particles are subject to Fermi's statistics.

In the case of Fermi's statistics no more than one particle may be contained in a cell of the phase space with a volume of h^s. In Bose's mechanics, on the contrary, each cell may contain any number of particles*).

An example of a gas subject to Fermi's statistics is the electronic gas; another instance is the gas of single ionized atoms of helium.

The constant C is determined in both cases by the given N and E. When N is given the constant C decreases, and as E decreases and for a sufficiently small E for those values of ε for which $Ce^{\frac{\varepsilon}{kT}} << 1$ we have:

$$f(q,p) = \frac{1}{h^s}, \qquad (6)$$

i.e. there is only one particle in each cell of the phase space with a volume h^s.

A gas which is in a condition such that the distribution of most of the particles in the phase space is described by formula (6) is called a degenerate gas.

On the contrary, if for a given N we take a very large E then in all regions of the phase space, except the small region where $\varepsilon(q, p)$ is very small, we may neglect unity in formulae (4) and (5) and consequently we obtain:

$$f(q,p) = \frac{1}{C h^s} e^{-\frac{\varepsilon(q,p)}{kT}}. \qquad (7)$$

The gas described by this limiting formula is called a classical gas. According to Quantum Mechanics, the particles forming a gas may not possess any amount of energy, generally speaking. The possible values of the energy $\varepsilon_1, \varepsilon_2, \varepsilon_3 \dots$ in case of a closed volume form always a discrete totality (obtained by solving Schrodinger's equation). In each stationary state the particle has one of the v values of energy ε_1. On the other hand, in each stationary state the impulses and the coordinates of the particles are indefinite. Therefore, each stationary state is characterized not by a point in the phase space of classical mechanics, but by some region. This region has a volume equal to h^s, therefore the numbers of par particles in each of these states must be equal to $f(q,p)h^s$.

---

*) There are indications that the dependence of the form of statistics on the even numbered charge breaks down in the realm of the physics of atomic nuclei.

We see, that according to Bose's statistics there are

$$\frac{1}{Ce^{\frac{\varepsilon_i}{kT}}-1}$$

atoms in each of the stationary states, and in the case subject to Fermi's statistics there are

$$\frac{1}{Ce^{\frac{\varepsilon_i}{kT}}+1}$$

atoms. In the case of the degenerated Fermi's gas formula (6) shows that in each stationary state there is only one particle.

We find for the classical statistics that in each stationary state there are $\frac{1}{C}e^{-\frac{\varepsilon_i}{kT}}$ atoms. This conclusion is expressed in the form of the following law.

Boltzmann's Principle. The numbers of atoms with energies $\varepsilon_1, \varepsilon_2, \varepsilon_3 \ldots$ are to each other as

$$g_1\, e^{-\frac{\varepsilon_1}{kT}}, \quad g_2\, e^{-\frac{\varepsilon_2}{kT}}.$$

where g_1, g_2, ... are integers expressing respectively the numbers of stationary states with energies $\varepsilon_1, \varepsilon_2 \ldots$

We shall consider the following special cases.

a) Suppose we have a single atomic gas upon which no outer force is acting. Then

$$\varepsilon = \frac{m\,(\dot{q}_1^2+\dot{q}_2^2+\dot{q}_3^2)}{2}, \qquad (8)$$

where $\dot{q}_1$, $\dot{q}_2$, $\dot{q}_3$ - are the components of the velocities and M is the mass. Formula (7) may then be reduced to

$$f(q,p) = Ce^{-\frac{m(\dot{q}_1^2+\dot{q}_2^2+\dot{q}_3^2)}{2kT}}, \qquad (9)$$

where C is some constant. This is Maxwell's Law of distribution of velocities.

Integrating, we find from condition (2)

$$C = \left(\frac{1}{2\pi mkT}\right)^{\frac{3}{2}}\frac{N}{V}. \qquad (10)$$

Integrating with respect to various directions of the velocity vector we see that the number of atoms dN, having velocities contained in the interval between v and v+ dv, is equal to

$$dN = 4\pi v^2\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} Ne^{-\frac{mv^2}{2kT}}\,dv. \qquad (11)$$

b) Suppose the gas is in a uniform field of gravitational force. Then

$$E = \frac{m(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2)}{2} + mgq_1. \qquad (12)$$

The fondamental equation (7) tells us that the density of atoms at an altitude q_1, after the integration with respect to velocities is equal to

$$\rho = \rho_0 e^{-\frac{mgq_1}{kT}}, \qquad (13)$$

where ρ_0 is the density at the altitude $q_1 = 0$.

c) The Law of Equal Distribution of Energy. It is easy to show in the case of classical statistics that the mean kinetic energy corresponding to one degree of freedom is always equal to $\frac{1}{2}kT$ and it does not depend on the nature of the particles of the gas.

Hence, in the case of a mono atomic gas when $s = 3$, the mean kinetic energy of each particle will be equal to $\frac{3}{2}kT$. It will thus remain same in mixture of mono-atomic gases.

Thus, if we shall denote by $\bar{\varepsilon}$ the mean kinetic energy of the particles of some monoatomic gas, then

$$\bar{\varepsilon} = \frac{3}{2} kT. \qquad (14)$$

§ 2. Equations of State. We may always compute the pressure for the given temperature and density using the knowledge of th distribution of particles in the phase space. The equation connecting pressure p, density ρ and temperature T is called the equation of state. For an ideal gas (consisting of particles which does not interact) the pressure is always equal to

$$p = \frac{2}{3} \frac{E_t}{V}, \qquad (15)$$

where E_t is the total energy of the progressive motion of all atoms of the gas, and V is its volume.

As the progressive motion has three degrees of freedom, E_t will be equal, according to classical statistics, to

$$E_t = \frac{3}{2} NkT,$$

whence

$$p = \frac{N}{V} kT. \qquad (16)$$

For the number of particles in a unit of volume we have

$$\frac{N}{V} = \frac{\rho}{m} = \rho \frac{\mu}{m} \frac{1}{\mu} = N_0 \frac{\rho}{\mu},$$

where μ is the molecular weight, and $N_0 = \frac{\mu}{m} = \text{const.}$ is Avogadro's number. Hence

$$p = kN_0 \frac{\rho T}{\mu}.$$

If we set $k N_0 = R$ (Klapeiron's constant),

then

$$p = \frac{R}{\mu}\rho T. \qquad (17)$$

This is the usual form of the equation of state of a classical ideal gas.

The equation of state for a Fermi gas is very complex in the general case. In the case of complete degeneracy (C is small)

$$p = \frac{1}{20}\left(\frac{3}{\pi}\right)^{\frac{2}{3}}\frac{h^2}{m^{\frac{8}{3}}}\rho^{\frac{5}{3}} \qquad (18)$$

or

$$p = K\rho^{\frac{5}{3}},$$

where K is constant. Because of (2) and (5) we may say that complete degeneration for a given constant temperature appears only for great densities (N is large). Therefore the equation of state (18) is applicable to Fermi's gas only for great densities.

§ 3. The Formula of Ionization. As was done in the case of a gas consisting of similar particles, it is possible to consider the most probable distribution for a gas consisting of particles of two or more types which may combine with each other. The following case is of great interest in astrophysics. We have a gas consisting of neutral atoms of some type (let us say of lithium), of electrons and of singly ionised atoms, i.e. atoms which have lost one electron. We shall call them hereafter simply ions. Ions may combine with electrons and form neutral atoms and, vice versa, neutral atoms may be decomposed into ions and electrons. Whenever an atom is ionized, i.e. when an eledtron is taken away from the atom, energy is expended. Let us denote the energy by χ. In a recombination, i.e. when the ion combines with an electron, the same quantity of energy χ is liberated. In any conceivable condition of a gas some atoms x N (where N is the total number of atoms) are ionized and the others (1-x) N are neutral. The fundamental problem arising in this case is the following one: what is the ratio of the number of ionized atoms to the number of neutral atoms in thermodynamic eqilibrium and at the temperature T? In other words to what is the quantity x/(1 - x) equal? The ansere to this question is given by the "formula of ionization".

$$\frac{x}{1-x}n_e = e^{-\frac{\chi}{kT}}\frac{(2\pi mkT)^{\frac{3}{2}}}{h^3}\frac{b'}{b}\frac{\sigma}{\sigma'}. \qquad (19)$$

Here n_e is the number of electrons in a unit of vloume. The quantity b is expressed by the formula:

$$b = g_1 + g_2 e^{-\frac{\varepsilon_2-\varepsilon_1}{kT}} + g_3 e^{-\frac{\varepsilon_3-\varepsilon_1}{kT}} + \dots \qquad (20)$$

where g_1, g_2, g_3, ... are the numbers showing how many stationary states correspond to the various values of $\varepsilon_1, \varepsilon_2, \ldots$ of the energy of the atoms; and here, unlike the previous exposition, we have in mind the inner energy of the atom; b' is the corresponding sum for an ionized atom.

In most cases in practice the differences $\varepsilon_2-\varepsilon_1, \varepsilon_3-\varepsilon_1, \ldots$ are large as compared with k T.

Therefore we may assume $\dfrac{b'}{b} = \dfrac{g_1'}{g_1}$,

where g_1' is the number of stationary conditions corresponding to the minimum value of energy of an ionized atom (the multiple of the fundamental condition of an ionized atom). The number σ is some "factor of symmetry" equal to the number of systems into which the terms of a given atom decompose (see a later section); σ'... is the corresponding number for an ionized atom.

If we denote the number of normal atoms in a unit of volume by n and the number of ionized atoms by n^+, then, since $n^+/n = x/(1-x)$:

$$\frac{n^+}{n} n_e = \frac{g_1'}{g_1} \frac{\sigma}{\sigma'} \frac{(2\pi m k T)^{\frac{3}{2}}}{h^3} e^{-\frac{\chi}{kT}}.$$

In the same way is determined the ratio of the number of n^{++} - twice ionized atoms - to n^+ - the number of singly ionized atoms:

$$\frac{n^{++}}{n_+} n_e = \frac{g_1''}{g_1'} \frac{\sigma'}{\sigma''} \frac{(2\pi m k T)^{\frac{3}{2}}}{h^3} e^{-\frac{\chi_1}{kT}}.$$

Here χ_1 is the energy of a singly ionized atom, g_1'' is the multiple of the fundamental state of a doubly ionized atom, and σ'' is the corresponding factor of symmetry.

According to quantum mechanics, $\dfrac{g_1}{\sigma_1} = 2$ for hydrogen atoms; for normal atoms of helium $\dfrac{g_1}{\sigma_1} = \dfrac{1}{2}$, etc. Hence, we may write the formulae of ionization in the form

$$\frac{n^+}{n} n_e = \frac{(2\pi m k T)^{\frac{3}{2}}}{h^3} e^{-\frac{\chi}{kT}}, \tag{21}$$

$$\frac{n^{++}}{n} n_e = \frac{(2\pi m k T)^{\frac{3}{2}}}{h^3} e^{-\frac{\chi_1}{kT}}, \tag{22}$$

if we neglect the influence of the factor $\frac{g_1}{\sigma_1}$ and $\frac{g_1'}{\sigma_1'}$ as compared with the exponential factor which varies greatly.

The ionization of atoms may occur under the influence of various external causes. The following two cases are of great interest to astrophysics:

1) the ionization of an atom takes place on account of an absorption by a neutral atom in the normal state of quantity of radiant energy greater than χ;

2) the ionization of an atom occurs during a collision of an atom with some particle (an atom, an electron etc); and the kinetic energy of the colliding particle and of the atom diminished transforming itself into the energy of ionization.

The ionization potential of a given atom is the potential V_i of the electric field in which the electron possesses the potential energy
The kinetic energy of an electron is also measured by the potential V of the field in which $eV = \frac{mv^2}{2}$. In other words the energy of the electrons is "measured" in volts. In order that an atom may be ionized by means of an electron collision, it is necessary that the kinetic energy of the electron be greater than $eV_i = \chi$. For instance, the ionization potential of helium is equal to 24.48 volts, and the ionization potential of He^+, i.e. of singly ionized helium, is equal to 54.2 volts. The ioniaation potential of doubly ionized silicon Si^{++} is equal to 33.35 volts etc.

4. The Laws of Radiation. a) Kirchhoff's Law. Let us investigate a body which is in thermodynamic equilibrium. Let us denote the quantity of radiant energy of frequency ν radiation by the body in the direction of the solid angle $d\omega$ in a unit of time and passing through the unit plane perpendicular to the axis of angle $d\omega$ through $E_\nu d\omega$. Next, let $K_\nu \, d\omega$ be the quantity of energy of frequency passing through the given body in the direction of the solid angle $d\omega$. From this quantity a portion will be absorbed by the body the amount being $A_\nu \cdot K_\nu \cdot d\omega$. According to Kirchhoff's Law the ratio

$$E_\nu / A_\nu = I_\nu \qquad (23)$$

is some universal function depending only upon the frequency and the temperature of the body and is equal to the intensity of the radiation of frequency ν within the volumn which is in thermodynamic equilibrium at the temperature T. The intensity of radiation of frequency ν is the coefficient I_ν In the expression

$$I_\nu \, d\nu \, d\omega \, dt.$$

$I_\nu \cdot d\nu \cdot d\omega \, dt$ represents the quantity of energy at frequencies between ν and $\nu + d\nu$ passing through a plane of $1 cm^2$ in the time dt, in the direction of solid angle $d\omega$ around the normal to this plane. It is easily seen that when we have a body absorbing the entire radiation i.e. $A_\nu = 1$, then it follows from (23) that $E_\nu = I_\nu$

= I_ν , i.e. I_ν is equal to the intensity of radiation of an absolutely black body.

It is required next to discover what this universal function I_ν is.

(B) Planck's Law. Planck's Law supplies the answer. The intensity of radiation I_ν inside a volume in thermodynamic equilibrium, or, which is the same thing, the intensity of radiation of an absolutely black body is

$$I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}, \qquad (24)$$

where C is the velocity of light.

The density of radiation of frequency ν i.e. the quantity of energy ($\rho_\nu d\nu$ in a unit of volume the frequencies of which are contained between ν and $\nu + d\nu$, and is expressed by $I_\nu d\nu$ as follows:

$$\rho_\nu \, d\nu = \frac{4\pi}{c} I_\nu \, d\nu. \qquad (25)$$

Hence

$$\rho_\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}}-1}. \qquad (26)$$

Substituting for frequency ν the wavelength $\lambda = \frac{c}{\nu}$ and $d\nu = -\frac{c}{\lambda^2}d\lambda$ for the intensity of the radiation I_λ we have

$$I_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{c_2}{\lambda T}}-1}. \qquad (27)$$

$I_\lambda d\lambda$ is the intensity of radiation the wavelength of which are contained between λ and $\lambda + d\lambda$. The constant c_2 has the value

$$c_2 = \frac{hc}{k}. \qquad (28)$$

c) Stefan - Boltzmann Law. Integrating expression (26) with respect to all frequencies we find the following for the density of the entire radiation

$$\rho = aT^4; \qquad (29)$$

where

$$a = \frac{8\pi^5 k^4}{15c^3h^3}. \qquad (29a)$$

We find for the total intensity of radiation the following by integrating (24) with respect to the frequencies

$$I = \frac{\sigma}{\pi} T^4, \qquad (30)$$

where

$$\sigma = \frac{2\pi^5}{15} \frac{k^4}{c^2 h^3} = \frac{c}{4} a. \qquad (30a)$$

Let us investigate the normal component

$$H = \int I \cos\theta \, d\omega,$$

of the flow of energy outward from the absolutely black body.

Here θ is the angle formed by the direction of radiation with the normal to the surface. As I, according to (30) does not depend on the direction

$$H = \pi I,$$

or

$$H = \sigma T^4. \qquad (30b)$$

Conversely, the temperature of an absolutely black body may be determined through H.

D) Wien's Law. The value λ_{max} of the wavelength at which I_λ has a maximum is connected.with the temperature by the following relation which is a result of differentiating (27)

$$\lambda_{max} T = .2 \cdot c_2, \qquad (31)$$

Hence, the maximum of the radiation is shifted in the direction of the short waves when the temperature is increases. Equation (31) is Wien's Law.

E) The Entropy of Radiation. The antropy of radiation is expressed by the temperature as follows:

$$\eta = \frac{4}{3} a V T^3; \qquad (32)$$

where V is the volume of the vessel in which the radiation is contained. On the other hand the entire energy of radiation E, according to (29), is equal to

$$E = \rho V = aVT^4 \tag{33}$$

Introduction (33) into (32)

$$\eta = \frac{4}{3}\sqrt[4]{aVE^3} \tag{34}$$

Let there be a volume V_1 in which radiation is contained at temperature T_1 and let there be a volume V_2 where there is no radiation. Let these two volumes be separated by a wall. When this wall is broken, the temperature of radiation in the new volume $V_1 + V_2$ will be lower than T_1 (as appears from (33)) if the radiation is to remain that of a black body and the energy E does not change. Also, according to (34), the entropy of radiation increases.

§ 5. Quanta of Light. Light Pressure. The electromagnetic radiation of matter may be broken up into separate monochromatic vibrations of various frequencies ν. The energy of the entire radiation is equal to the sum of the energies of the separate monochromatic vibrations. The energy of each monochromatic vibration ε is in turn equal to an integral factor of the quantity of energy $h\nu$ where h - is Planck's constant.

$$\varepsilon = nh\nu \qquad (n = 0, 1, 2, \ldots)$$

The quantity of energy $h\nu$ bears the name of the "light quantum" or photon. Thus we may say that the electro-magnetic field consists of photons. Matter may absorb and radiate only an integral number of photons.

Each photon of energy $h\nu$ has an impulse the magnitude of which is equal to $h\nu/c$ and the direction coincides with the direction of the wave vector of the given monochromatic vibration. In the capture of the quantum by matter the impulse of the quanta is transfered to the matter. Consequently, whenever the energy of the radiation E, proceeding in the given direction is absorbed, the matter receives an impulse E/C in the same direction.

Under conditions of thermodynamic equilibrium the radiation is isotropic, i.e. the intensity of radiation is the same in all directions. Therefore, the impulses obtained by matter through radiation flowing in the given directions naturally cancel each other and the resulting impulse is equal to zero.

In the absence of strict thermodynamic equilibrium a different situation will occur. Let us take, in a field of radiation where the intensity I_ν is a function of the direction, and elementary rectangular parallelipiped dS · dh the dimensions of the base of which are large as compared with the altitude dh. Let us investigate the normal to the plane dS. The normal component of the

radiation I dω cos θ passing in a unit of time through the parallelipiped in a direction which forms an angle θ, with the normal and which is contained within the solid angle dω, is equal to $\frac{Id\omega}{c} = \cos^2\theta \cdot dS$. A part of this impulse, namely the impulse of the absorbed energy $\frac{Id\omega}{c} \cos^2\theta \cdot dS \cdot dh \cdot \sec\theta$, where the quantity α is called the volume coefficient of absorption, is transmitted to the matter contained in the volume dS·dh. Thus, this volume will get the impulse

$$\alpha\, dh\, dS \frac{Id\omega}{c} \cos\theta .$$

in the direction of the normal. The resulting impulse obtained from the radiation flowing in all directions is

$$\frac{\alpha\, dh\, dS}{c} \int I \cos\theta \cdot d\omega = \frac{\alpha\, dh\, dS}{c} H_h, \qquad (35)$$

where H_h is the flow of radiation in the direction of the normal towards dS.

$$H_h = \int I \cos\theta\, d\omega . \qquad (36)$$

In this formula the integration is extended to all solid angles dω.

H_h is the normal component of some vector $\vec{H}_c$ with the components H_x H_y, H_z where

$$H_x = \int I \cos\alpha\, d\omega ; \quad H_y = \int I \cos\beta\, d\omega ; \quad H_z = \int I \cos\gamma\, d\omega \qquad (37).$$

where α, β and γ are angles made by the direction of radiation respectively with the axes x, y, z.

Of the radiation I cos θ · dS·dω, falling upon the given parallelipiped in a unit of time, and flowing in the given direction, a quantity α·I·cos θ·dS·dω·ds is absorbed along the path ds = sec θ·dh. Besides, the same parallelipiped radiates in the same direction a certain quantity of energy equal to αE·dS·dh·dω Therefore, we have the following for the changes in this path

$$d(I\cos\theta)\, d\omega\, dS = -\alpha I \cos\theta\, dS\, ds\, d\omega + \alpha E \cos\theta\, dS\, ds\, d\omega$$

or

$$\frac{dI}{ds} = -\alpha I + \alpha E . \qquad (38)$$

It is easily seen from this equation that vector $\vec{H}$ may be represented as a diverging of a certain tensor of tensions with the components:

$$\left.\begin{array}{ll} p_{xx} = \frac{1}{c}\int I \cos^2\alpha\, d\omega , & p_{xy} = \frac{1}{c}\int I \cos\alpha \cos\beta\, d\omega \\ p_{yx} = \frac{1}{c}\int I \cos\beta \cos\alpha\, d\omega , & p_{yy} = \frac{1}{c}\int I \cos^2\beta\, d\omega , \\ p_{zx} = \frac{1}{c}\int I \cos\gamma \cos\alpha\, d\omega , & p_{zy} = \frac{1}{c}\int I \cos\gamma \cos\beta\, d\omega , \end{array}\right\} \qquad (39)$$

(See p. 13)

$$\left.\begin{aligned} p_{xz} &= \frac{1}{c}\int I\cos\alpha\cos\gamma\,d\omega, \\ p_{yz} &= \frac{1}{c}\int I\cos\beta\cos\gamma\,d\omega, \\ p_{zz} &= \frac{1}{c}\int I\cos^2\gamma\,d\omega. \end{aligned}\right\} \qquad (39)$$

Indeed, we may rewrite (38) in the form

$$\frac{dI}{dx}\cdot\frac{dx}{ds}+\frac{dI}{dy}\cdot\frac{dy}{ds}+\frac{dI}{dz}\cdot\frac{dz}{ds}=-\alpha I+\alpha E,$$

but

$$\frac{dx}{ds}=\cos\alpha; \quad \frac{dy}{ds}=\cos\beta; \quad \frac{dz}{ds}=\cos\gamma;$$

whence

$$\cos\alpha\frac{dI}{dx}+\cos\beta\frac{dI}{dy}+\cos\gamma\frac{dI}{dz}=-\alpha I+\alpha E.$$

Multiplying each in turn by the $\cos\alpha$, $\cos\beta$, $\cos\gamma$ and integrating with respect to ω and noticing that $\int E\cos\alpha\,d\omega = 0$ as the radiation is equal in all directions, we obtain, according to (37) and (39) the following system of equalities:

$$\left.\begin{aligned} \frac{dp_{xx}}{dx}+\frac{dp_{xy}}{dy}+\frac{dp_{xz}}{dz} &= -\frac{\alpha}{c}H_x \\ \frac{dp_{yx}}{dx}+\frac{dp_{yy}}{dy}+\frac{dp_{yz}}{dz} &= -\frac{\alpha}{c}H_y \\ \frac{dp_{zx}}{dx}+\frac{dp_{zy}}{dy}+\frac{dp_{zz}}{dz} &= -\frac{\alpha}{c}H_z \end{aligned}\right\} \qquad (40)$$

The tensor of tensions p is called the tensor of light pressure. For isotropic radiation, it follows from (5) that

$$p_{xy}=p_{yz}=p_{xz}=p_{yx}=p_{zy}=p_{zx}=0$$

$$p_{xx}=p_{yy}=p_{zz}=\frac{1}{3c}\int I\,d\omega=\frac{4\pi}{3c}I=p. \qquad (41)$$

These diagonal components of the tensor are called in this case, simply "the light pressure". As the density of radiation ρ is equal to $\frac{4\pi}{c}I$,

$$p=\frac{1}{3}\rho=\frac{1}{3}aT^4. \qquad (42)$$

The light pressure for thermodynamic equilibrium is equal to 1/3 of the density of the radiant energy. If the intensity of radiation at a given direction depends upon but one coordinate z and but one angle γ then, in this case (which is close to reality in stellar atmospheres) all the non-diagonal elements will be equal to zero, and for p_{zz} we shall have the equation.

$$\frac{dp_{zz}}{dz} = -\frac{a}{c} H_z. \qquad (43)$$

In order to compute the quantities p_{xx} and p_{yy}, we should use the equality

$$p_{xx} + p_{yy} + p_{zz} = p,$$

which is true under all conditions.

Formula (42) may be applied for the deep layers of stars and also, approximately, in the layers close to the surface.

Thus, we see that the mechanical action of radiation upon the absorbing medium appears in the form of tensions. These tensions enter the usual hydrodynamic and hydrostatic equations according to the rule of hydrodynamics.

§ 6. Doppler's Principle. Let us consider an atom having some velocity $\vec{V}_1$ and an inner energy ε_1. Let this atom emit a photon in the direction which forms an angle θ with the direction of the vector. The inner energy now receives a new value ε_2. The photon has an impulse $\vec{P}$ the direction of the velocity vector which is equal to $h\nu/c$. In order that the law of the conservation of energy may remain in force it is necessary that the atom obtain after the radiation an additional umpulse $\vec{P}$ which will cause a small change in velocity. Let us denote the changed velocity by $\vec{V}_2$. If we denote the mass of the atom by m, then applying the laws of energy and of impulse we may write

$$\varepsilon_1 + \frac{m}{2}\vec{V}_1^2 = \varepsilon_2 + \frac{m}{2}\vec{V}_2^2 + h\nu,$$

$$m\vec{V}_1 = m\vec{V}_2 + \vec{P}.$$

Taking $\mathcal{E}_1 - \mathcal{E}_2 = h\nu_0$ where ν_0 is the frequency of radiation of a static atom and squaring the second equation we may write

$$\frac{m}{2}(\vec{V}_1^{\,2} - \vec{V}_2^{\,2}) = h(\nu - \nu_0),$$

$$m^2 \vec{V}_1^{\,2} = m_2^2 \vec{V}_2^{\,2} + 2m\frac{h\nu}{c}|\vec{V}_2| \cos\theta + \frac{h^2\nu^2}{c^2}.$$

Neglecting the quantity which is inversely proportional to the square of the velocity of light and comparing these two qualities, we have

$$h(\nu - \nu_0) = \frac{h\nu}{c}|\vec{V}_2| \cos\theta$$

or taking $\nu - \nu_0 = \Delta\nu$

$$\frac{\Delta\nu}{\nu} = \frac{|\vec{V}_2| \cos\theta}{c}.$$

For the change of the photon's wavelength λ connected with ν by the relation $\lambda = c/\nu$ we have

$$\frac{\Delta\lambda}{\lambda} = -\frac{|\vec{V}_2| \cos\theta}{c} = -\frac{V_{2r}}{c} \qquad (44)$$

where V_{2r} is the component of the velocity of the atom in the direction of the extension of the quantum of light. Should we imagine on the path of the photon an observer with a spectral apparatus, V_{2r} will represent the velocity of the atom as the atom approaches the observer. When the atom moves away from the observer V_{2r} will be negative.

We see that as the source of light approaches the observer the wavelength diminishes, and if the source of light moves away the wavelength increases.

We neglected the effects of relativity in this deduction.

§7. Spectroscopic Terms. According to quantum mechanics atoms and molecules may be in various stationary states. The various stationary states of some p particles are differentiated by indices or by a group of indices. We shall distinguish them at present only by one index. In each stationary state the object under consideration (molecule, atom, ionized atoms, electron etc.) has a definite energy E_n. If we have $E_n > En'$ for two conditions n and n', the object under consideration may pass from condition n into condition n', radiating energy equal to $En - En'$ with a frequency $\nu_{nn'}$ determined by the condition

$$E_n - E_{n'} = h\nu_{nn'},$$

where h is Planck's constant. This quantity of energy, as we know, is called a photon or a quantum of light. A reversed transfer from n' ⟶ n is possible when quantum $h\nu_{nn'}$ is absorbed.

Thus all frequencies radiated and absorbed by the given object may be represented as a difference of numbers

$$T_n = -\frac{E_n}{h},$$

which bear the same of spectroscopic terms. Thus

$$\nu_{nn'} = T_{n'} - T_n .$$

If T_n' and T_n are discrete terms, i.e. if none of them is the point of ~~cont~~ contact of condensation of a group of terms, the frequency $\nu_{nn'}$ is isolated from other frequencies of the given object and we have a spectral line. If the collection of terms T_n contains a part which entirely fills some interval, the index n becomes in this interval continuous, and the frequencies

$$\nu_{nn'} = T_{n'} - T_n$$

form a continuous spectrum.

Transformations from one state into another may take place not only as a ~~re~~ result of radiation or absorption of a single quantum, but also under the influence of other outer disturbances, as for instance, collisions with other atoms, electrons etc.

Also possible are transformations accompanied by radiation (or absorption) of two or more quanta, the sum of the frequencies of which is equal to $\nu_{nn'}$. However, the probability of such transfers is very small, and we must take it into consideration only when the transfer with the radiation of one quantum is impossible.

§ 8. The Spectrum of Hydrogen like Atoms. The spectrum of hydrogen or hydrogen-like ions (He+, Li++, etc) is the simplest. Thus, for instance, the hydrogen atom has the following possible values of energy.

$$E_n = -\frac{2\pi^2 \mu e^4}{h^2} \frac{1}{n^2} \quad (n = 1, 2, 3, \ldots) \text{ and } E > 0,$$

where e is an electronic charge, and μ is expressed by the mass of the nucleus M and by the mass of the electron m as follows:

$$\frac{1}{\mu} = \frac{1}{M} = \frac{1}{m},$$

i.e. μ differs little from m. Thus the entire collection of values of the energy consists of a discrete negative part and of a continuous positive part. The number n, representing the indicator number of the discrete state, is called the total quantum number. If we let

$$R = \frac{2\pi^2 \mu e^4}{h^3},$$

then

$$T_n = \frac{R}{h^2}$$

and the entire collection of the spectral lines of hydrogen has the frequencies

$$\nu_{nm} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \quad \begin{array}{l} m = 1, 2, 3, \ldots. \\ n = m+1, m+2, m+3, \end{array}$$

The series of lines obtained by the way of transfers from all the higher states ($n > 1$) into the first is called Lyman's series. The frequencies of these series are obtained if we set m = 1.

$$\nu_{n1} = R\left(1 - \frac{1}{n^2}\right).$$

For m = 2 Balmer's series is obtained

$$\nu_{n2} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

for $m = 3$ Paschen's series is obtained etc. As $n \to \infty$ these series approach lim limits called the head of the series, the frequencies of which are R, $R/4$, $R/9$, etc., respectively. If E > 0 for the upper condition i.e. if it is ionized, then

for the transfers into condition $m = 1$ various frequencies occur which are larger than R. Thus, the continuous spectrum is always attached from the violet side to the head of the spectral series. The same applies also to other series of hydrogen and in general to all elements.

The transfers of atoms from condition $E < 0$ into condition $E > 0$ are processes of ionization as, for $E > 0$, the electron is no longer connected with the atom. Reversed transfers are recombinations.

If Z is the charge of a nucleus of a hydrogen-like atom, the only difference from the structure of the Hydrogen spectrum is that the discrete terms have the value

$$T_n = \frac{RZ^2}{n^2}$$

and hence all frquencies are equal to the hydrogen frequencies multiplied by Z^2.

Thus, in the case of ionized helium all wavelengths of the lines are four times less than those of corresponding lines of hydrogen. For $m = 4$ we obtain Pickering's series of ionized helium.

$$\nu_{n4} = 4R\left(\frac{1}{4^2} - \frac{1}{n^2}\right) = R\left(\frac{1}{2^2} - \frac{1}{(\frac{n}{2})}\right).$$

It is easy to show that when n is an even number of these frequencies coincide with those of the Balmer series. The very slight difference is due to the difference in R i.e. in μ. Thus in the Pickering series He^{+}, every second term of the series, coincides with a term of the Balmer series.

Each term of hydrogen $\frac{R}{n^2}$ corresponds not to a single stationary condition, but to a whole series of them. The difference between their energies is determined by relativity corrections which we neglected: Each term having the main quantum number n corresponds to $2n^2$ conditions. They differ among themselves by the different wave function that characterizes them. These conditions may be renumbered with the aid of indices l, j, m,. Index j characterizes the total moment of momentum of tha atom, including the spin of the electron, index m is called the (magnetic) polar quantum number, and $l + 1$ is called the azimuthal quantum number. For a given main quantum number n, the quantum number l assumes the values

$$0, 1, 2, \ldots\ n-1$$

The conditions and terms characterized by these terms l are denoted respectively by the letters

$$S, P, D, F, \ldots$$

The value of the main quantum number is set to the left of the letter characterizing l, and the value of the general moment of momentum of the atom is set to the right below. Thus the symbol 4 $P_{3/2}$ means the term for which n = 4: l = 1; j = 3/2.

In the hydrogen and hydrogen-like atoms for a given the number j may take two values: $l + 1/2$ and $l - 1/2$

For giving values of l and j, m may take the following values:

$$-l, -l+1, \ldots -1, 0, 1, \ldots\ l-1, l$$

i.e. $2l + 1$ values altogether.

As j may take two values for a given l, the number of possible pairs of j and m will be $2(2l + 1)$. Summarizing with respect to all possible values of l we find that for a given n there are altogether $2n^2$ stationary states of an atom. In order to characterize some transfer, the first and last terms are given, for instance

$$4P_{3/2} - 2S_{1/2}.$$

The spectra of alkaline metals or of similar ions (Ca+, for instance) have structure similar to that of hydrogen. However, whereas in the hydrogen-like atoms the terms of the same main quantum numbers are so close in their level of energy that spectrographs of great dispersion are required to distinguish them, in the alkaline metals the difference of two terms of equal n but of different l may reach the same order as the difference of two terms of different n.

Thus, for instance, the terms 3 $D_{5/2}$ and 3 $D_{3/2}$ of Ca+ lie between the terms 1 $S_{1/2}$ and 2 $P_{3/2}$. However, the difference between the terms of equal main quantum numbers approaches zero as the quantum number increases. The distance between the terms having the same n and but different j is small as compared with the differences of terms having various l for a given n.

As j has two values for the given n and l two terms close to each other obrain which form the so called relativity doublet. When l = 0, there may be cases when j = 1/2 and j = - 1/2. However, in both cases we write S 1/2, as the magnitude of the term depends solely on the absolute value of j. In order to denote the doublet character of the term, the number 2 is written on the upper left side of the letters S, P, D .... For instance, $2^2 P_{3/2}$ and $2^2 P_{1/2}$ of atom Ca+ from a doublet. Whey they pass into $1^2 S_{1/2}$

$$1^2S_{1/2} - 2^2P_{3/2},\quad 1^2S_{1/2} - 2^2P_{1/2}$$

the well known H and K lines of the solar spectrum appear. The lines H and K of calcium and the lines of other elements corresponding to a possible transformation from a normal state (with the least energy) into the nearest condition in the energy scale are called resonance lines, and in the case of alkali metals are called resonance doublets. As was shown above, in the case of calcium the term 3 D is nearer to 1 S than to 2 P. However, the transformations 3 D - 1 S accompanied by the discharge of quanta are "forbidden", i.e. they are of small probability as compared, for instance, with the transfers 1 S - S P.

§ 9. Rules of Selection: Metastability. We have seen that not all transfers between any two pairs of conditions of the atom are possible. Some are of so small a probability that they are called "forbidden". There are some rules of selection which enable us to distinguish possible transfers from "forbidden" ones. The most important rules are:

1) The azymuthal quantum number or must at each transfer change by unity on account of radiation and absorption. In other words:

$$\Delta l = \pm 1$$

Therefore, transfers of the type nS - n'P, nP - n'D etc. are possible, but transfers of the type nS - n'F, nP - n'F, nS - nD etc. are forbidden.

2) The quantum number j may change or not change by unity, i.e.

$$\Delta j = 0, \pm 1.$$

For instance, the transfer $3^2 D_{3/2} - 2^2 P_{1/2}$ is possible, but the transfer of $3^2 D_{5/2} - 2^2 P_{1/2}$ is forbidden.

In most cases the "forbidden" is of a conditional character, i.e. the corresponding transfers are of small probability only as compared with those of large probability. Only in a few instances the transfer with the discharge or capture of one quantum is impossible, and two quanta must be radiated or absorbed. We shall not enter here into details. In the basic quantum condition (with the smallest energy) the atom remains a long time when outer disturbances are rearranged. The usual average life of an atom in an excited condition is of the order order 10^{-8} sec or 10^{-7} sec. After an interval of time of the order mentioned the atom spontaneously passes into conditions of lower energy and it radiates a photon. If, however, all transfers from the given excited condition are forbidden" the life of the atom in such a condition may exceed the usual one. Such conditions of excititations are called metastable conditions. The previously mentioned terms for Ca+, namely, $3^2 D_{5/2}$ and $3^2 D_{5/2}$ and $3^2 D_{3/2}$ may serve as an example of metastable conditions. Below these terms lie only the term 1^2 S however, according to the rules of selection a transfer into this condition is "forbidden".

Likewise, the condition 2S 1/2 of hydrogen is metastable as the transfer into the fundamental condition 1 $S_{1/2}$ of hydrogen from this level is forbidden, and all the other levels are higher or of the same level.

Of great importance for astrophysics is the problem of the life of atoms in various metastable conditions, or, which is the same thing, the problem of probabilities of "forbidden" transfers. Unfortunately, the experimental and theoretical data in this respect are few. It was established experimentally that in some cases the life of an atom reaches 10^2 sec. However, it is certain that metastable conditions exist in which an atom enjoys a long life, especially those conditions which are possible only by means of radiation of the quanta.

§ 10. Spectral series of alkaline metals and of similar ions. The totality of lines obtained from transfers of various conditions into conditions $1^2\ S_{1/2}$ is called the main series.

The common term of this series, has the form

$$1^2S_{1/2} - m^2P_{1/2\,,\,3/2} \qquad (m = 2, 3, \ldots),$$

and for j = 1/2 and j = 3/2 two components of a doublet are obtained. The totality for lines obtained from transfers of various D - conditions into one of conditions 2 P is called the first diffuse series.

Its common term has the term

$$2^2P - m^2S \qquad (m = 3, 4, \ldots.)$$

The possibile values of j in the common terms may be easily pre ieved from the rule of selection. The totality of lines

$$2^2P - m^2S \qquad (m = 3, 4, \ldots.)$$

is called the second diffuse series.

Lines

$$3^2D - m^2F$$

constitute Bergmann's series.

§ 11. Spectrum of Helium and of Earth-Alkaline Metals. The totality of the terms of helium is divided into two systems of terms. One system consists of triplets (orthohelium) the other consists of singlets (par-helium). The main quantum number n and the symbol l may take in each system of terms the same

system of values as those of the alkaline metals, except that the term of 1 S which exists only in the system of singlets.

For a given l the possible values of j are seen from the following tables:

System of Singlets

j l		0	1	2	3	4
S	0	0				
P	1		1			
D	2			2		
F	3				3	
G	4					4

System of Triplets

j l		0	1	2	3	4	5
S	0		1				
P	1	0	1	2			
D	2		1	2	3		
F	3			2	3	4	
G	4				3	4	5

The following terms correspond to the main quantum number let us say, 3, and to $l = 2$ in the system of triplets: $3^3 D_1$, $3^3 D_2$, $3^3 D_3$, which together form a triplet term. In the system of singlets only the term $3^1 D_2$ corresponds to these quantum numbers. The rules of selection within the system of terms of perhelium (singlets) and of orthohelium (triplets) remain the same as for the alkaline metals. As to the intercombinations between the terms of parhelium and of orthohelium, they are "forbidden". Among the intercombination lines the line 1 S - $2^3 P_1$ has been observed. Evidently there is a greater probability of transfer in this line than for other lines of intercombination.

The condition 2^1 S and 2^3 S, are metastable as transfers from them to a lower energy level 1^1S are forbidden. The second of them is especially very metastable, for the transfers $2^3 S_1 - 1^1$ S are forbidden according to the rule of selection $\Delta l = \pm 1$ as well as on account of the prohibition of intercombinations.

In the presence of an external electric field many forbidden lines of helium were observed. Among them the line 2^3 P - 4^3 F with a wavelenght = 4470A is situated beside the permitted line 2^3 P - 4^3 D whose λ = 4472Å.

The spectra of the earth-alkaline metals have a structure similar to that

of the spectrum of helium. We shall note here two important differences:

1) The rule of prohibition between the terms of the system of singlet and triplets is less strict than in the case of helium (the corresponding probabilities of transfer are greater).

2) The distances between the terms of the same triplet exceed by scores of times those of helium where they are so small that the differences in wavelenght are about 1/2 A° only (this fact is the reason why they cannot be resolved on stellar spectrograms).

The spectra of the elements which follow in the periodic system are of yet greater complexity than the spectrum of helium. Their investigation is beyond the present introduction.

§ 12. Spectra of Molecules. The spectra of molecules are of a more complex structure than that of the atoms. We shall say here only a few words about them.

The condition of molecules, aside from the quantum number determining the condition of the outer electron, as in the case of the atom, is characterized by additional quantum numbers describing the condition of the atoms forming the molecule.

For instance, for the molecule of two atoms there are: 1) The vibration quantum number and 2) the rotational quantum number.

The vibfational quantum number characterized the vibration of atoms forming the molecule about the position of equilibrium, and the rotational quantum number describes the rotation of a molecule as that of a solid body.

The terms of the same electron quantum numbers and of the same vibrational quantum number but of a different rotational quantum number (but of a different rotational quantum number) are so close together that the spectral lines corresponding to the transfers between two groups of similar terms form the so called molecular band which can be resolved into lines only with spectrogrphs of comparatively large dispersion.

The bands corresponding to different pairs of vibrational quantum numbers, but to the same transfer of an electron form a system of bands. A separate sy system of molecular bands corresponds to each type of transfer of an electron.

§ 13. The Einsteinian Coefficients of Transfer. In the absence of an outer radiation each atom in some condition n may spontaneously pass by emitting a

quantum of energy into condition n' having less energy. According to Einstein the frequency of such transfers is subject to the following law: if at the moment t there are N_n atoms in condition n, then during the time dt, $N_n A^n_{n'} dt$ atoms will pass into condition n'. "The coefficient of probability" $A^n_{n'}$ depends only upon the conditions n and n'. If there are several conditions n' having less energy than n, the number of atoms which pass spontaneously from condition n into all other conditions will be determined by the expression

$$\frac{dN_n}{dt} = -N_n \sum_{n'} A^n_{n'} dt = -N_n A_n dt,$$

where $A_n = \sum A^n_{n'}$ is the coefficient of probability of the transfer into any condition.

Thus

$$N_n = (N_{n0}) e^{-A_n t},$$

whence it is easy to see that the average life of the atom in condition n is

$$T_n = \frac{1}{A_n}.$$

As we have already said, T_n for the usual non-metastable conditions reaches the order of 10^{-8} sec, hence $A_n \approx 10^{-8}$ $\sec^{-1}$. The frequency radiated in the transfer n → n' is either equal to or close to $\nu_{nn'} = \frac{E_n - E_{n'}}{h}$.
For each interval of wavelength between ν and $\nu + d\nu$ there is some probability $j_{nn'}(\nu)d\nu$ that in the transfer n → n' the radiation frequency will lie in that interval. Evidently $\int_0 j_{nn'}(\nu)d\nu = 1$. The following formula developed by Wigner and Weisskopf holds for some of the simplest transfers

$$j_{nn'} = \frac{1}{4\pi^2} \frac{\Gamma_{nn'}}{(\nu + \nu_{nn'})^2 + \left(\frac{\Gamma_{nn'}}{4\pi}\right)^2},$$

Where $\Gamma_{nn'} = A_n + A_{n'}$. However, this formula was derived for the case where the field of radiation is absent at the moment t = 0. This formula may be used, for instance, for an excitation by means of an electronic collision. In case of an optical excitation the behavior of function $j_{nn'}(\nu)$ depends on the condition of excitation.

The transfers under the influence of radiation are possible along with the spontaneous transformations. Let the density of the radiation in frequencies

contained in the interval between ν and $\nu+d\nu$ be equal to $\rho_\nu d\nu$. Then for each pair of conditions n > n' there is some function $k_{n'}^{n}(\nu)$ such that the product

$$k_{n'}^{n}(\nu)\rho_\nu dt$$

is the probability that the atom will pass during the time d t under the influence of radiation from state n into state n' emitting a quantum of energy of frequency ν . Conversely

$$k_{n'}^{n'}(\nu)\,\rho_\nu dt$$

is the probability that the atom will pass from state n' into state n, absorbing the quantum ν If ρ_ν is constant; within that range of frequencies where $k_{n'}^{n}$ and $k_{n}^{n'}$ have a value noticeably different from zero, (within the spectral line), then the probability of transfer under the influence of radiation is represented in the form

$$B_{n'}^{n}\rho_\nu dt \quad and \quad B_{n}^{n'}\rho_\nu dt,$$

respectively, where

$$B_{n'}^{n}=\int_0^\infty k_{n'}^{n}(\nu)d\nu \quad and \quad B_{n}^{n'}=\int_0^\infty k_{n}^{n'}(\nu)\,d\nu.$$

The quantity $B_{n}^{n'}$ is called the Einsteinian coefficient of absorption and $B_{n'}^{n}$ is called the Einsteinian coefficient of negative absorption. The following relations obtain for the two given conditions n and n', whose weitht is g_n and $g_{n'}$ respectively, between the coefficients $A_{n'}^{n}$, $B_{n'}^{n}$ and $B_{n}^{n'}$.

$$B_{n}^{n'}=\frac{g_n}{g_{n'}}B_{n'}^{n}; \qquad A_{n'}^{n}=\frac{8\pi h\nu^3}{c^3}B_{n'}^{n}.$$

The following relation obtains between the density of radiation ρ_ν and the intensity of radiation I_ν for isotropic radiation:

$$\rho_\nu=\frac{4\pi}{c}I_\nu.$$

Therefore the probability of transfer n' → n under the influence of radiation of intensity I_ν , passing perpendicularly to the area 1cm^2 in the solid angle dω, will be equal to

$$\frac{1}{c}{}_\nu k_{n'}^{n}(\nu)\,I_\nu\,dt\,d\omega.$$

The mathematical expectancy of the energy absorbed is equal to this probability multiplied by the quantity of energy of the quantum i.e. as the quantity of energy passing through the surface in the time dt will be $I_\nu dt d\omega$:

$$\frac{h\nu_{nn'}}{c} \cdot k^{n}_{n'}(\nu)\, I_\nu\, dt\, d\omega.$$

The ratio of these quantities, i.e. the percentage of the absorbed energy with respect to the total energy passing through a unit of area, is called the coefficient of absorption of a single atom:

$$\alpha_\nu = \frac{h\nu_{nn'}}{c} k^{n}_{n'}(\nu).$$

If the line is not widened by various effects and has the so called "natural" width determined by the diminishing of radiation, the quantity α_ν for resonance lines is of the order of 10^{-11}. When the lines widens this quantity becomes smaller as may be shown from the following considerations.

By the definition of $k^{n}_{n'}$ and $B^{n}_{n'}$ we have $B^{n}_{n'} = \int_{0}^{\infty} k^{n}_{n'}(\nu)\, d\nu.$

If within the section of frequencies $\Delta\nu$ in which $k^{n}_{n'}$ has values no smaller than, let us say, one third of the maximum value, then, denoting the mean value $k^{n}_{n'}$ by $\bar{k}^{n}_{n'}$ we have approximately

$$\bar{B}^{n}_{n'} = \bar{k}^{n}_{n'} \Delta\nu$$

and for the mean value of $\bar{\alpha}_\nu$ in this section we have

$$\bar{\alpha}_\nu = \frac{h\nu_{nn'}}{c} \frac{B^{n}_{n'}}{\Delta\nu}$$

whence it is apparent that as the line widens α_ν decreases.

The expression of function α_ν depends in the general case upon the method of excitation. The following formula may be applied to the resonance lines and to some other cases:

$$\alpha_\nu = \frac{c^2}{32\pi^2\nu^2} \frac{g_n}{g_{n'}} \frac{A_{nm}\, \Gamma_{nn'}}{(\nu - \nu_0)^2 + \left(\frac{\Gamma_{nn'}}{4\pi}\right)^2}.$$

§ 14. The Expression of the Einsteinian Coefficients by means of Matrix Elements. Sun Rule. If we denote by $q_{nn'}$ the matrix element of coordinate q when the matrix element is diagonal, *) then, according to quantum mechanics, we shall have the following expression for the coefficient of spontaneous transfer**)

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*) Here $q_{nn'}$ is a vector whose components are $x_{nn'}$, $y_{nn'}$, $z_{nn'}$.

* *) Here only the two-field radiation is taken into consideration. Therefore, according to this formula the probability of trnasfer of the type S-D is equal to zero, although in reality it differs slightly from zero.

For $B_n^{n'}$ we have

$$B_n^{n'} = \frac{8}{3}\pi^3 \frac{e^2}{h^2} |q_{nn'}|^2.$$

These formulae obtain only when the weight of each condition n and n' is equal to unity. Otherwise, more complicated expressions are used which we shall omit here.

With the aid of these formulae the probabilities of transfer for hydrogen and the approximate values of these probabilities for some lines of more complex elements were computed.

Let us write Heisenberg's expression for the relation of transposition in its developed form for the diagonal elements.

$$\sum_{n'} (p_{nn'} q_{n'n} - q_{nn'} p_{n'n}) = \frac{3h}{2\pi i}^{*}.$$

as

$$p_{nn'} = 2\pi i \overline{\nu}_{nn'} m q_{nn'}, \text{ where } \nu_{nn'} = \frac{E_n - E_{n'}}{h},$$

and where m is the mass of an electron, then

$$\sum_{n'} \overline{\nu}_{nn'} |q_{nn'}^2| = -\frac{3h}{8\pi^2 m}.$$

When n > n' then $\overline{\nu}_{nn'} > 0$ and it is equal to the usual frequency $\nu_{nn'}$ and when n < n' we have $\overline{\nu}_{nn'} = -\nu_{n'n}$'n. Hence

$$\sum_{n'>n} \nu_{nn'} |q_{nn'}|^2 - \sum_{n'<n} \nu_{nn'} |q_{nn'}|^2 = \frac{3h}{8\pi^2 m}.$$

The usual notation is

$$f_{nn'} = \frac{8\pi^2 m \nu_{nn'}}{3h} |q_{nn'}|^2.$$

The quantities $f_{nn'}$ and $A_{nn'}$ are connected by the following relation:

$$f_{nn'} = \frac{1}{8\pi^2} \frac{mc^3}{e^2 \nu_{nn'}^2} A_{nn'},$$

as is apparent from the representations of both quantities by matrix elements.

Evidently

$$\sum_{n'>n} f_{nn'} - \sum_{n'<n} f_{nn'} = 1.$$

This equality is usually called "Sum Rule". Whence from n = 1

$$\sum_{n'=2}^{\infty} f_{nn'} = 1.$$

Therefore, the maximum value f for any transfer from the fundamental condition may not be greater than unity. Hence

$$A_1^{n'} \leqq 8\pi^2 \frac{e^2 \nu_{n'}^2}{mc^3}.$$

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*) Here $p_{nn'}$ as well as $q_{nn'}$ is a vector with three components. The product is understood to be scalar.

Astrophysical observations often enable us to determine the product $A_{1n'}N$, where N is the number of atoms in the first condition in the atmosphere of a star (in a column of 1 cm^2 cross-section). As the previous formula gives the upper limit for $A_1^{n'}$, we may always determine the lower limit of N.

§ 15. *Perturbations of Terms.* When outside perturbations take place the terms change their magnitude, and terms which usually coincided now deverge. A split of multiple terms takes place. A split occurring as a result of a magnetic field (Zeeman effect), for instance, differs greatly from one which appears as a result of an electric field (Stark effect). If the electric field is of a high degree of heterogeneity, each atom of the given element forms its own image of the split of terms. When put one on top of the other, the spectra of such a collection of atoms form one spectrum with widened lines*).

Let us investigate two terms Tn' and Tn". Let them be, respectively, the multiples of gn, and gn". When perturbations appear let these terms split into simple terms which differ by a new quantum number m. We obtain two collections of terms Tn'm' and Tn"m".

If we neglect the quantities which are proportional to the first degree of the perturbation, the following relation exists for the probabilities of transfers of $A_{n''}^{n'}$ and of $A_{n''m''}^{n'm'}$ before and after the perturbation takes place:

$$g_{n''} A_{n''}^{n'} = \sum_{m'm''} A_{n''m''}^{n'm'},$$

This relation is called the *rule of spectroscopic stability.*

Unfortunately, starting out from this rule it is only indirectly possible to establish the relation between the intensity of the line in the absence of perturbations and the sum of intensities in a split line. Only for thin radiating layers observed in terrestrial laboratories does the rule of spectroscopic stability hold as follows: *The intensity of a line in the absence of perturbations is equal to the sum of the intensities of the separate components into which the spectral line splits in the disturbing field.*

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*) A more detailed account of the Zeeman and Stark effects may be found in: Frisch's Atomic Spectra, 1934.

2. Refractors and Reflectors

by I. A. Balanovsky

The attainments of astrophysics to which we all bear witness depend not only upon the discovery of new methods of research but also, in a very large measure, upon the continuous improvement of instruments which the astronomer must use in his study of the heavenly bodies from this or that point of view. The instruments used in such investigations - spectrographs, photometers etc. -- are usually attached to the telescope which collects in its focal plane a large quantity of light and keeps the image of the object under investigation in the same field of view by means of special machanisms. Therefore, we must reconize the telescope as the fundamental instrument of astrophysical investigations.

Telescopes may be divided into refractors and reflectors according to the kind of optical system producing the real image of the object under investigation. In the refractors objectives consisting of two or more lenses are used for this purpose while in the reflectors the image is formed by means of a concave parabolic mirror. Evidently in both cases the larger tha aperture of the instrument the brighter will be the image obtained. This ability to collect light is the most important one in the telescope for the purposes of astrophysics. An increase in the effective aperture enables us to study the fainter celestial objects and to penetrate deeper into the universe. However, it would be wrong to assume that this ability of the telescope - its light gathering power - is entirely dependent upon the eimensions of the objective or mirror. The real light gathereing power depends upon a series of causes among which the most important are diffraction and various aberrations.

§16. The Role of Diffraction in the Formation of the Focal Image.

Because of the tremendous distances that separate the heavenly bodies from the earth, the rays of light falling on the objective or mirror may be considered as parallel. In this case the rays collect at the principal focus of the objective or mirror. On account of diffraction the image of the shining point will appear on the focal plane in the form of a disk of noticeable diameter and surrounded by rings the brightness of which decreases as their size increases.

It is possible on the basis of the wave theory of light to compute accurately the distribution of brightness in the focal image of the bright point.

If we denote by r the radius of the objective or mirror, the brightness at any point of the focal plane is determined by the expression:

$$I = C\pi r^4 \varphi^2;$$

where φ represents the following expression

$$\varphi = \frac{\pi}{4}\left(1 - \frac{1}{2}\frac{n^2}{2} + \frac{1}{3}\frac{n^4}{(2.4)^2} - \frac{1}{4}\frac{n^6}{(2.4.6)^2} + \dots\right).$$

For the sake of brevity we take

$$n = \frac{2\pi}{\lambda} \frac{s}{F} r. \qquad (45)$$

where s is the distance in the focal plane between the point and the optical axis, F is the focal distance, λ the wavelength of light, C is a constant depending upon the brightness of the source of light as well as upon the transparency of the glass of the objective or the reflective ability of the mirror.

The value of function φ may be easily computed for a series of values of n and then we may have a fair knowledge of the distribution of brightness in the focal plane. These values are given in Table 1 (second column).

The third column of the table gives the brightness, and it is assumed that the brightness for a point on the optical axis is equal to unity. Studying this table we conclude that the focal images of the stars, which we condiser as shining points because of their tremendous distances, are represented as disks surrounded by a series of rings. The central disk containes about 5/6 of the entire light. The brightness of the first diffraction ring is 2°/o of the central disk. The second ring is only 0.4°/o.

Let us find the angular radius δ of the little circle of the diffractional image of brightness I. As in seconds of arc

$$\delta = \frac{s}{F \sin 1''} \qquad (45')$$

we obtain from (45) and (45'):

$$\delta = \frac{n\alpha}{D}, \qquad (46)$$

where D is the diameter of the objective and

$$\alpha = \frac{\pi}{\lambda \sin 1''}.$$

If we take for n the value corresponding to the first dark ring (I = 0) the formula (46) will evidently give us the diameter of the central disk. This diameter is inversely proportional to the aperture of the telescope. As the aperture is doubled the diameter of the diffraction disk is halved.

It is not difficult to see that for the stars the degree of concentration of light at the focus will increase as the diameter of the diffraction disk diminishes. Hence the light gathereing power of telescopes will be in this case proportional to the fourth power of the apertures. However, because of the peculiarities of the structure of the ratina of the human eye during visual observation or because of the structure of the sensitive layer of a photographic plate during photographic observations, the light gathering power of telescopes may be taken as proportional merely to the square of the diameter;

$$H = CD^2.$$

For those objects which occupy some definite area on the sky (planets, comets, nebulae) the degree of concentration of light will increase as the focal length of the objective or mirror decreases. In this case the theoretical light gathering power will be

$$H = C\left(\frac{D}{F}\right)^2.$$

§ 17. The Resolving Power of Telescopes. We have already seen that the first dark diffractional ring is obtained for n = 3.8. Substituting this value into formula (46) and taking such values of the wavelength as correspond to the greatest sensitivity of the eye and of the photographic plate, namely: $\lambda = 570\mu\mu$ and $\lambda = 430\mu\mu$, we obtain the diameter of the diffraction disk:

$$\delta_{vis} = \frac{14''.0}{D}; \qquad \delta_{ph} = \frac{10''.7}{D}.$$

Table 1

n		φ	I	n		φ	I
0.0	+	1.0000	1.0000	4.4	-	0.0922	0.0085
4	+	0.9801	0.9606	8	-	1244	0155
8	+	9221	8502	5.2	-	1320	0174
1.2	+	8305	6897	6	-	1194	0142
6	+	7124	5075	6.0	-	0922	0085
2.0	+	5767	3326	4	-	0568	0032
4	+	4335	1879	8	-	0192	0004
8	+	2927	0857	7.2	+	0151	0002
3.2	+	1633	0265	6	+	0419	0017
6	+	0530	0028	8.0	+	0587	0034
4.0	-	0330	0011	4	+	0645	0041

In these formulae D is expressed in centimeters.

Now we may obrain an idea of the resolving power of telescopes. Let a double star be observed whose components are at a distance ρ'' from each other. According to what was said previously each of the stars will be represented by a disk the diameter of which is given by the formulae already stated. Evidently, we shall see both stars as entirely separated only if $\rho'' > \delta''$. This condition is the characteristic of the resolving power of a telescope. It is higher for photographic refractors or astrographs than for the visual refractor having the same aperture. In reality, however, the theoretical resolving power is never attained which fact is explained by the imperfection of the optical system and the harmful influence of the unstable atmosphere. In photographic observations an additional impediment for the attainment of the theoretical resolving power is the granular structure of the sensitive layers of the photographic plate.

§ 18. The Main Defects of Objective Lenses and Mirrors. The previous discourse had reference to an ideal optical system having no defects whatever. In reality the optical system has a series of defects which greatly disturb the ideal dåffractional picture to which reference was made previously. These defects are in part a result of the defects in the glass of which the objective or mirror is made and in part a result of the process of forming of the image of the real object by the optical system. All efferts of modern optical the technology are turned towards overcoming these defects by improving the quality of optical glass so that it may have all the necessary qualities for the preparation of faultless objectives and by repeated construction of better optical systems having a minimum of aberrations and other faults. Much progress has been made in this field although there is still much remaining to be done.

The most important defects of objective lenses and mirrors influencing their light gathering power or image quality and resolving power are as follows:

a) Spherical Aberration. This depends on the fact that rays falling upon various zones of the objective lens or concave mirror are not collected in one point, that is, the extreme rays are collected farther from the focus than the central rays. Therefore the image of the shining point is surrounded by a more or less bright aureole. Spherical aberration diminishes as the focal length of the objective lens or mirror increases, but, on the other hand, it increases as the aperture becomes larger. By giving to the lenses a proper curvature it is possible to diminish aberration greatly, but a residual aberration always remains, increasing by several times the diameter of the diffraction disk and diminishing thereby the light-gathering power of the refractor. As to the reflectors, they are free from spherical aberration in the optical axis if they have the shape of a paraboloid of rotation.

b) Chromatic aberration is the most troublesome property of visual and photographic refractors and it occurs because there is no combination of lenses of various radii of curvature and of various kinds of glass that would bring to one point all the rays of various wavelengths. Therefore, it is endeavoured in accordance with the particular kinds of refractors to bring to a common focus those rays to which either our eyes or the photographic plate are most sensitive. In visual objectives the rays of 480 - 600μμ in wavelength are usually collected. For astrographs rays of 400 - 450μμ are usually selected. Other rays would be collected either further or nearer than the common focus.

One of the characteristics of the quality of an objective is its chromatic curve. The latter is constructed as follows: the wavelengths appear as abscissae and the corresponding deviation of the focus from the minimum as ordinates. As an example data are given in Table 2 for the three instruments of the Pulkovo Observatory: the 30 inch refractor, the normal astrograph and the Bredichin astrograph.

Here the deviation of the given rays from the minimum is expressed in fraction of the focal length. The deviation in millimeters would be expressed by F. Δ F. A comparison of the chromatic curves of the normal and of the Bredichin astrographs shows the advantages of an objective of many lenses of the Bredichin astrograph type over an objective of two lenses of the normal astrograph type in regard to improvement in chromatic aberration.

Chromatic aberration greatly diminishes the theoretical light gathering power of refractors since the objective collects at its focus only those rays which lie near the apex of the chromatic curve. The rest of the rays scatter forming circles outside the focus of greater and greater diameter as the distance from the apex of the curve increases.

The mirror of the reflector collects at its focus all the rays falling on it regardless of wavelength. This is the great advantage of reflectors over the refractors. Having the same aperture the reflector has greater light gathering power than the refractor because reflectors have no chromatic aberration.

Some further defects in the images formed by objective lenses and by mirrors take place as the image is moved away from the optical axis. They are as follows: coma astigmatism, and field curvature.

Coma and astigmatism are distortions of the form of the image of a shining point lying off of the optical axis. On account of the coma the image of the point takes a comet-like form and it has a very uneven distribution of brightness. Astigmatism is manifiested by the stars which are far form the optical axis taking a cross-like form with a bright center. An example of images distorted by coma and by astigmatism is given on Fig. 1a and b. *

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* Many half tone illustrations appearing in the original were impossible to copy with the desired degree of accuracy and the reader is referred to the original text.

Both of these faults are of great importance in reflectors where the field of non-distorted images does not generally exceed 30' in diameter when a parabolic mirror is used. It is theoretically possible, according to Schwarzschild, to construct a reflector which would have a more useful field. However, the so-called anaberrational systems are not in general use as yet because of the difficulty of manufacturing the necessary mirrors which are of very complex form. Only in the last few years the German optician Schmidt, using an original idea, has been able to construct a reflector of great light gathering power giving astonishingly clear photographs of portions of the sky up to 10° in diameter. As to refractors, they are more fortunate in this respect. The present day photographic objectives of astrographs are able to cover a non-distorted field up to 10° in diameter.

The focal surface i.e. the surface upon which the rays converge best is in general not plane but usually concave towards the objective. This circumstance is important for photographic refractors where the plane photographic plate is set tangent to focal surface. The point of contact is called the optical center of the plate. The images of stars situated far from this center are more or less outside of the focus. If the focal surface is close to that of a sphere, the linear distances of the stellar images from the optical center of the plate will evidently be proportional to the tangents of the corresponding angular distances when the optical axis of the objective passes through the optical center of the plate. This property is called the normal distortion of the objective. In reality the focal surface differs always more or less from a spherical surface, and as a result an uneven distribution on the plane occurs which circumstance is called the anomalous distortion. An effort is always made to reduce the latter in objectives built for accurate astronomical work. The curvature of the field becomes greater as the focal length of the objective lens or mirror becomes smaller. We shall say subsequently more about distortion when we shall speak about the special propreties of photographic objectives.

§ 19. The Loss of Light in Refractors and Reflectors. Besides the imperfect convergence of rays at the focus, the light gathering power of telescopes is diminished also because of absorption and reflection of light by the lenses. Let us denote the intensity of the light falling upon the surface of the lense by I. If the index of refraction is n, then according to Fresnal the surface of the lens will reflect the following amount of light:

$$I_o = I\left(\frac{n-1}{n+1}\right)^2.$$

The quantity of light passing through the lens will be $I_1 = I - I_o$. When the light comes out of the lens another reflection will take plade, and hence the amount of light coming out of the lens will be:

$$I\left[\frac{4n}{(n+1)^2}\right]^2.$$

Besides the loss of light on account of reflection light is also lost because of the absorption of lingt by the lens. If we denote the coefficient of absorption of glass by α and the thickness of the lens by d, the quantity of light that will pass through it will be (Lambert's formula):

$$I' = I\alpha^{d}.$$

The total loss of light as a result of reflection and absorption gives the final intensity as follows:

$$I'' = I\alpha^{d}\left[\frac{4n}{(n+1)^2}\right]^2.$$

We shall have for several lenses the following expression:

$$I'' = I\alpha_1^{d_1}\alpha_2^{d_2}\dots\left[\frac{4n_1}{(n_1+1)^2}\right]^2\left[\frac{4n_2}{(n_2+1)^2}\right]^2\dots$$

By this formula it is possible to compute the loss of light in a given objective if the indices of refraction, the coefficients of the absorption in glass, and the thickness of the lenses are known. The loss on account of reflection for an average objective of two lenses is about 20°/o and that for an objective of four lenses is about 30°/o. The total loss of light in an average objective of two lenses is about 50°/o.

Table 2

30 - Inch Refractor F = 14.1m		Normal Astrograph D = 0.33m F = 3.6m		Bredichin Astrograph D = 0.17m F = 0.8	
λ in $\mu\mu$	ΔF	λ — in $\mu\mu$	ΔF	λ in $\mu\mu$	ΔF
360		360	0.00125	360	0.00200
400		400	014	400	40
440		440	019	440	0
480	0.00445	480	100	480	50
520	108	520	210	520	100
560	40	560	325	560	165
600	0	600	430	600	230
640	100 100	640	538	640	300
680	390	680			

In regard to the loss of light the reflector has many advantages over the refractor. The mirror of the former, covered with a thin layer of silver, concentrates at its focus about 90°/o of light that falls on the mirror. The percentage is somewhat different for rays of different wavelength. The reflective ability of the silver layers diminishes as time passes. It is therefore necessary to renew the silver layer from time to time.

§ 20. Types of Objectives. A simultaneous satisfaction of all the conditions needed for the elimination of all the faults of optical system is a very difficult problem of practical optics, and this problem can be solved only approximately. Depending on the purpose of the instrument, a few faults are selected which are eliminated almost entirely while all the other faults are eliminated as much as possible.

The simplest type is the ordinary visual objective consisting in most cases of two lenses: one of which collects the rays and is made of crown glass, and the other of which scatters the rays and is made of flintglass. Such a system corrects the spherical and chromatic observations for images lying close to the optical axis. The remaining chromatic aberration, the so called secondary spectrum, may be practically reduced to zero in objectives of three lenses of the apochromatic type. Such objectives, however, are used only in refractors of comparatively small dimensions on account of their large weight and the difficulty of manufacture.

Of considerably great difficulty are the design and production of photographic objectives with a large useful field. These objectives require correction for coma and astigmatism. In photographic objectives of two lenses the diameter of the field suitable for accurate astrophotographic measurements does not exceed $1^{1}/2°$. This is not sufficient for some photometric investigations and problems of a statistical character.

In order to obtain a large field of undistorted images and to correct the aberrations it is necessary to construct complex objectives consisting of three four and more lenses. The improvement of astrophotographic objectives has proceeded parallel to the development of photographic optics. New kinds of glass in addition to thelông known crown and flint glass become necessary. A search for new kinds of optical glass began in the middle of the 19-th century, but only toward the end of the last century was a new assortment of optical glass obtained dur mainly to the systematic efforts of O. Schott of Jena. These new kinds of glass satisfy most of the requirements of a modern astrophotographic objective*). Of the numerous types of astrophotographic objectives we shall mention here only a few which have found the most wide spread application.

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*) In pre-revolutionary times almost all optical glass, especially that for astromical purposes was imported from abroad. At present there are establishments in the USSR which make the most complex optical instruments and optical glass.

First of all we must mention the doublet of the Petzval type built for the first time in 1841 of old types of glass. The objectives of this type consist of two pairs of lenses divided by a large interspace. With a light gathering - power of 1/5 they give a well corrected field of about 4° in diameter (Fig. 2a, page 45).* In 1895 the English optician Taylor developed an objective of three lenses two collecting lenses and one diverging lens situated between the other two (Fig. 2b). Such objectives made by the Cook firm are called triplets and are distinguished by an excellent correction of the faults of field with a large light gathering power. To the triplet type also belongs Zeiss's Tessar (Fig. 2c, page 45) which differs from the other triplets by having a complex rear lens. The ratio of the diameter to the focal length in astrotriplets reaches 1/4.5 and the diameter of the field with a small light gathering - power reaches7°. To the objectives of greatest light gathering power belong also the recently built systems "Ernostar" and "Tachar". The ratio of D:F may be reduced to 1/1.5, but they are inferior to the other triplets with respect to aberrations, astigmatism and coma. Nevertheless, they are widely applied in special fields of astrophysics where it is necessary to obtain a large section of the Sky upon a single plate. Ross's system of objectives satisfies a great deal better the requirements of a large field with t the faults well corrected. Such requirements are insistently requested by those who are engaged in accurate astrometry.

Ross's objectives consist of two collecting and two diverging lenses(Fig. 2d), and with a comparatively small light gathering power they give a field of about 100 sq. degrees. The peculiarities of these objective consist in very sharp images of stars over the entire field and in an almost complete absence of anomalous distortion. Thus, the objectives made by Zeiss for the so called "zone" astrograph at Pulkovo the anomolous distortion does not exceed 1" at the edges of the plate which is 20 x 20cm.

§ 21. Systems of Reflectors. There are several systems of reflectors of which the systems of Newton and of Cassegrain are almost exclusively, used at present. In both of these types the main collector of light is a parabolic mirror which is made of glass and whose parabolic surface is covered with a thin layer of silver.

In the Newtonian reflector the conical pencil of rays from the main mirror is intercepted near its principal focus by a small plane mirror placed at an angle of 45° to the optical axis. Thus the real image of the object under observation is obtained at the side of the tube and just outside of it (Fig. 3a, page 46).

In the Cassegrain reflector a convex hyperbolic mirror is placed on the path of the rays reflected from the main mirror. This hyperbolic mirror is of a small diameter and it enlarges the image of the object on the optical axis. A hole is usually bored in the center of the large mirror so that the pencil of rays from the hyperbolic mirror may pass through it to the eye of the oberever. It is possible to avoid this construction by inserting a small plane mirror which would intercept the pencil of rays from the hyperbolic mirror and turn it aside to the outside of the type (Fig. 3b).

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* Page references are, in most cases, as given in the original text.

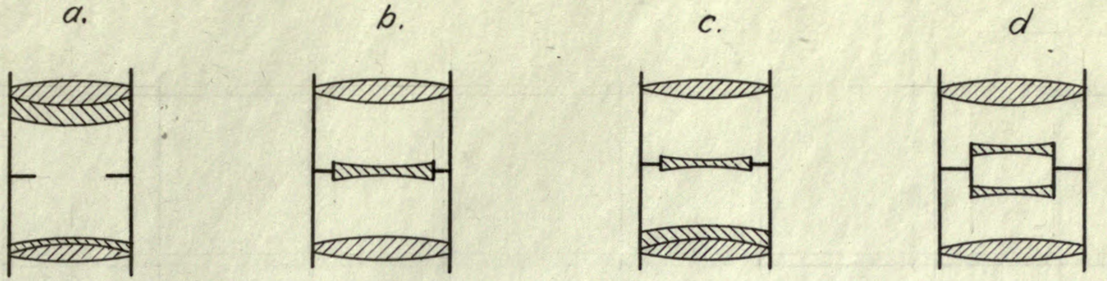


Fig. 2.

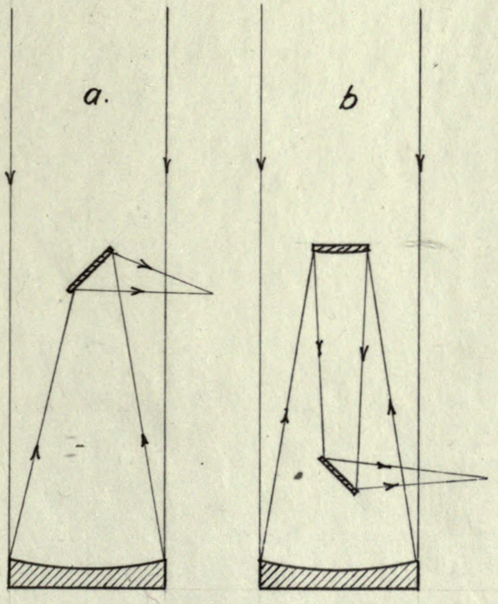


Fig. 3

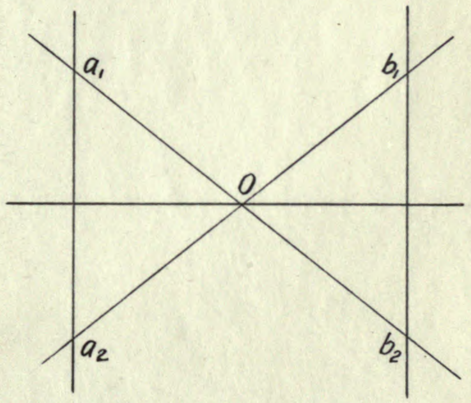


Fig. 4

Because of the hyperbolic mirror it is possible to correct the defects of the image of the main mirror and to obtain a somewhat larger field of undistorted images.

Reflectors are often so constructed that it is possible to change them easily from the Cassegrainion system of the Newtonian and vice-verse. This is done by changing their upper parts consisting of either a plane or a hyperbolic mirror or both.

§ 22. Investigation of Objective Lenses and Mirrors. A very simple and convenient method was proposed by I Hartmann for the investigation of the residual faults of spherical and chromatic observations and of astigmatism.

The objective of a refractor or the entering aperture of the tube of the reflector is covered by a screen, bearing small opening distributed in such a way, that sets of four of these openings, placed symmetrically relative to the center correspond to different concentric zones of the objective or the speculum mirror. Pointing the instrument to some bright star it is possible to obtain an out-of-focus photograph of such a screen. Let us place a photographic plate first in front of the focus and then behind it and let two sections of the focal scale A_1 and A_2 correspond to those positions. Then some two symmetric holes of the screen produce images at points a_1, a_2, and b_1, b_2 (Fig. 4). Let us measure upon a plate the distances $a_1, a_2 = e_1$ and $b_1, b_2 = e_2$. It is evident from the figure that the sections of the scale corresponding to focus O will be as follows:

$$A = A_1 + \frac{e_1}{e_1 + e_2}(A_2 - A_1). \qquad (47)$$

Applying this formula to any pair of holes of the screen we obtain a series of values A_φ^r which may be vary for zones of various radii r as well as for the position angles, φ, of the given pair of holes. The dependence of the focal position on r shows that the objective is not free from spherical aberration or of zonal faults. Its dependence on φ is determined by astigmatism. The latter is manifiested by the rays, lying at some section of the pencil of rays corresponding to angle $\varphi = \varphi_0$, having the smallest focus whereas for another section of $\varphi = \varphi_0 +$ 90 the focus will be largest. Therefore we may write for a a given pair of holes:

$$A_\varphi^r = A_{\varphi_0}^r + \alpha \sin^2(\varphi - \varphi_0).$$

For the other pair of holes placed perpendicularly to the first pair

$$A_{\varphi+90°}^r = A_{\varphi_0}^r + \alpha \cos^2(\varphi - \varphi_0).$$

The mean

$$\frac{1}{2}\left(A_\varphi^r + A_{\varphi+90°}^r\right) = A^r$$

will be free of astigmatism and it will give the position of the focus for the zone of radius r. Finally,

$$A = \frac{\Sigma r A^r}{\Sigma r}$$

determines the mean position of the focus for the entire objective.

Forming the difference $A^r - A_1$ we may compute the diameter of the image of the shining point inasmuch as it depends on spherical aberration. It is expressed in 0.00001 parts of the focal length as follows:

$$T = \frac{200\,000}{F^2} \frac{\Sigma r^2 (A^r - A)}{\Sigma r},$$

or in seconds of arc:

$$\delta = 2''.06\,T.$$

The quantity T is called the technical constant or Hartmann's constant, and it characterizes the quality of an objective or a mirror; their quality is higher the smaller T is. Of the large objectives the 40 - inch objective of the Yerkes Observatory, is considered the best. Its values of T is equal to 0.17. For a normal astrograph at the Pulkovo Observatory T = 0.106.

In order to investigate chromatic aberration it is most convenient to make use of a spectrograph attached to the refractor. Let us cover the objective by a screen having two holes which are symmetrical with respect to the center and which are perpendicualr to the slit of the spectrograph. Let us photograph the spectrum of a bright star at two positions of the slit of the spectrograph with respect to the focus of the objective: in front of the focus and behind the focus. Then the plate will show two pairs of spectra. The spectra of the first pair will be concave to one another, the spectra of the other pair will be concave to one another. The measurement of the distances of the spectra for various wavelengths will enable us to compute the positions of the focus for these wavelengths By formula (47).

§ 23. Mounting of Refractors and Reflectors. The optical power of a telescope may be fully made of use only if the mounting of the telescope is stable and convenient. The tube of telescope is usually fastened to a system of two movable axes which enable us to direct the telescope towards any point on the celestial sphere. One of these axes, the one which is situated in the plane of the meridian making an angle with the horizontal plane, equal to the latitude of the plane is called the hour axis or the polar axis. The other axis, perpendicular to the first is called the declination axis. The tube is fastened to the latter axis. Such a mounting is called parallactic mounting or an equatorial mounting. A very

important part of such a mounting is the clock mechanism which transmits to the polar axis a uniform motion, and the clock mechanism is so regulated that a complete rotation of the axis is made during a sideral day. Then the observed object will automatically remain in the same field of view of a telescope. Both axes carry graduated circles which serve for the setting of the telescope towards the given object by its right ascension and declination.

There are several types of equatorial mountings. Fig. 5 shows the 30-inch refractor of the Pulkovo Observatory mounted by Repsold according to the so called German system. Its axes are fastened to a high column of cast-iron placed upon a massive stone foundation. The defect of this system is the necessity of resetting of the instrument for observations at eastern and at western hour-angles. In the case of photographic observations the column is in the way of prolonged exposures near the meridian. To avoid this a curved column is used. The normal astrograph at Pulkovo (see Fig. 10) is an example of such mounting.

The same defect is avoided in the mounting of the so called English type where the polar axis is set on two stone columns. This mounting, in turn, has the defect that it does not enable the observer to observe objects close to the pole at all hour angles. The 72-inch reflector at the Victoria Observatory (Canada) (see Fig. 6) is an example of such mounting.

Due to the properties of the optical system of reflectors they are mounted differently. At the upper end of the polar axis is placed the so called fork inside of which rotates a cylindrical box with the main mirror. The 40-inch reflector at Simeis is mounted in this manner. (see Fig. 7).

3. Introduction to Astrophotography

by S. K. Kostinsky

§ 24. Advantages of Photography. History. The application of photography is based on the peculiar properties and advantages of the "photographic eye" -- as we call the photographic camera which is built in principle like the human eye. Indeed, the inner part of the eyeball corresponds to the camera obscura, the cornea plus the crystalline lens and the vitreous humor correspond to the compound photographic objective, the pupil of the eye corresponds to thediaphragm, and the retina is similar to the sensitive layer of the photographic plate. Finally the very process of seeing is determined by the photo-chemical action upon the visual purple which corresponds to the action of light upon the sensitive chemical combinations.

What are then the advantages that the photographic eye has above the human eye? We may define them by the words: 1) swiftness, 2) documentation, 3)integration; let us analyze them separately.

1) Swiftness. Our eye receives impressions rather slowly and they disappear not all at-once, but in their rapid sequence a coalescence of impressions takes place (the cinematograph principle.) Looking at a panorama we must observe it gradually, focusing various points for its clear seeing. On the contrary, a photographic eye sees everything at-once, with strong illumination, sometimes only in a $0^s.0001$ and it has the ability to resolve very rapid phenomena into their elements. This fact enables us to study by the aid of photography such phenomena as the structure of lightøning,the flight of birds, the motion of bullets and of canon - balls, the fall of bodies, etc. In its applications to astrononomy this property of the photographic eye, i.e. speed and simultaneous seeing -- exactly at the same moment -- of several neighboring objects (for instance, a double star, a comparison star and a planet, a star cluster etc) is of great importance, since it makes the observation entirely differential which cannot be said of visual observations. An objective study of such a brief phenomenon as the "solar corona" become possible only with the application of photography.

2) Documentation. With contemporary, greatly improved photographic objectives, a photograph gives us an accurate central projection of the panorama on the plane of the plate; two photographs taken from different points enable us to construct the exact form of the object photographed or the position of various points of the panarama in space. Hence, photographical negatives are quite unbiased documents which do not depend upon personal errors and which may be saved in an archive using them from various points of view. For instance, we photograph some region of the sky for the measurement of the relative position of the stars

contained therein; but we may make use of this photograph subsequently for the determination of the brightness of these starw as well as for the search for some new objects discovered several years later and not noticed previously.

3) Integration. This property is best expressed by the phase: our eye whether aided or not "the longer it looks the less it sees", because it gets tired. On the contrary, the photographic eye in many cases "the longer it looks the more it sees", because the sensitive photographic plate integratez the weak impressions of light. This fact is of tremendous importance for astrophotography, for it enables us to prolonging the exposition and increasing the sensitivity of the plate to obtain photographs of the faintest celestial objects and even of such objects as are invisible with the most powerful telescopes (the faint satellites of Jupiter and Saturn, faint nebulae, some comets near aphelion etc). We may add that the photographic eye is sensitive to such rays of the spectrum as the infrared and the ultra-violet rays up to the γ (gamma) rays which are not directly accessible to the human eye.

Let us say a few words about the history of photography and its development. This history is of interest also because astrophotography, i.e. the application of the photographic method of observations to practical astronomy, developed parallel to photography. It follows from the previously given definition of the latter consists of two parts: the optical part which teaches us how to construct correct images of photographed objects by projecting them upon the sensitive plane, and the chemical part divided into the negative and positive processes in which we acquaint ourselves with the action of radiant energy upon various substances and chemical combinations and with the chemical methods of fixation this action. In the middle ages and later in the eighteenth and the beginning of the nineteenth centuries the action of light on the substance used now is photography -- namely, on silver chloride (AgCl) -- had been already discovered and investigated; and it was thereby noticed that the various rays of the spectrum blacken the substance unevenly, a conception was formed of two kinds of rays: actinic and non-actinic (red) of which the latter do not act on silver cloride (Senebier in 1782, Ritter in 1801, Berard in 1812). In this period they knew how to construct a sufficiently good image in the camera obscura but their opticc were still very crude.

Only in the first half of the nineteenth century were serious investigations and discoveries made which led to modern photography. We must note the names of three men -- two Frenchman and one Englishman -- who had done most in this field and who may be truthfully called the "fathers of photography"; J. N. Niepce, 1765 1833, L. J. Daguerre, 1787-1851, and Fox Talbot, 1800-1877.

The first of them -- Niepce -- was first to obtain a positive and quite fixed image (i.e. an image which does not change by the action of light); he

also obtained the first etched clinhe on a metallic plate and thus laid the bases of heliogravure (1824). Daguerre, who worked together with Neipce from 1829 on and who improved the optical part of the photocamera, made a notable discovery (after Niepce's death) of the so-called "hidden image" on silver iodine which he ("developed" as we say now) by the vapours of mercury (1835). This discovery was of tremendous importance, for it made possible the shortening of the exposure down to a few minutes whereas Niepce used several hours in full sunlight. It became possible to photograph animate objects (portraits-daguerreotypes) and it was not for nothing that the famous astronomer F. Arago in his report to the French Acadamy of Sciences (August 19, 1839) propheisied the great application of photography in the future in all science including astronomy. As the first attainment, daguerreotypes of the Moon and Sun, and even bright stars (Draper, Bond, Fizeau, Foucault etc) were soon obtained. But in spite of the clearness of detail on the daguerrotypes, they soon disappeared from general use especially after F. Talbot made the following great step in the improvement of photography. The daguerrotype is a positive image on a metallic plate, and a new photograph is required in order to obtain a second copy. Talbot offered to use the negative (i.e. the reverse image) which he obtained on a simi-transparent paper made sensitive by silver chloride with addition of some silver bromide and silver nitrate. At first he used the visible image (blackening), later he used the hidden image producing it by the pyrogallic developer similar to the one used at present and he stabilized it by pyposulphite upon the advice of John Herschel (1841).

From the second half of the nineteenth century up to the present day the imporvement of photography has gone rapidly forward. In 1848 the paper negatives of Talbot were replaced by glass plates covered by some suitable substance such as albumen starch etc. which were made sensitive by the haloid salts of silver and became the carriers of photographic processes (Niepce de St. Victor). In 1851 the famous wet collodion process (Legray, Archer, Fry) was developed which is used even at the present day especially in photomechanical processes. This method, in spite of some inconveniences (the impossibility of a long exposure) had found application in astrophotography: excellent photographs of the Moon and Sun were obtained (Warren de la Rue, 1852, and others) and even special instruments -- the photoheliographs -- were built for systematic solar observations (in Kew, England and Vilno, formerly Russia, now Poland). Instruments of such a type were applied to the photographying of the transit of Venus over the solar disk in 1874. In America photographs of stars were obtained which were suitable for accurate measurements (Bond and his co-workers in Cambridge, in 1857 and Rutherford in 1864); this was the beginning of photographic astromentry, i.e. the application of photography to astrometric problems.

Of the next stages in the development of photography we shall mention the following: the discovery of the bromogelatine emulsion by Maddox in 1871, the discovery of a method to increase its sensitivity by the so called "ripening"

(Bennet, 1878) and also with the aid of ammonia (van Monckhoven, 1879), and finally, the industrial production of highly sensitive dry bromogelatine plate beginning in the eighties of the last century. The latter attainment had so simplified photographic operations that even children were enabled to play with it. The application of photography to numerous fields of Science and especially to astrography immediately followed: the astrographic labors of Hill, Common, Huggins, E. Pickering, Janssen and of the Henri bros. in the period led, finally, to the grand idea of making an atlas and catalogue of stars of the entire stellar heavens by photography. (see p. 86, part I in Russian).

§ 25. Optical Part of Photography. The main apparatus for the construction of an accurate photographic image is the photocamera consisting of three main parts: the camera-obscura, the objective, and the plate holder which contains the plate and which is fastened to the camera from the side away from the objective -- instead of the opaque glass with the aid of which the focusing is made. Necessary conditions for every photocamera are: 1) complete opacity and absence of light in the camera-obscura and in the plate holder, 2) the possibility of changing the distance between the objective and the opaque glass (or the plate holder) in order to bring the surface of the plate into the main, or conjugate focus of the objective, and 3) the centering of the objective so that its optical axis be perpendicular to the surface of the inserted plate.

Cameras are of various sizes and construction. We shall speak here only about astrophotographic cameras which consist in most cases of an ordinary astronomical telescope with a constant parallactic mounting -- in order that the diurnal motion of the heavens may be more conveniently observed -- but which are equipped with special photographic objectives, i.e. such objectives that collect, in the main, actinic rays. The peculiarity of these cameras as well as of their plate holders is that they are almost exclusively made of metal which is necessary in accurate work in order to avoid deformation of the apparatus of humidity (in the open air). Often such cameras are united with visual astronomic insturments and then a photographic refractor (or reflector) or the so called "astrograph" is formed.

We shall not enter here into the minute details and contrivances of astrophotographic cameras; we shall merely mention the instantaneous shutters (snapshots). These shutters placed before the objective or before the plate or immovably joined to the objective (between two Gauss points) are used when the length of the exposure is to last only a fraction of a second ($< 0\overset{s}{.}2$) as in the case of photographing the Sun or the Moon. They consist essertially of one or several opaque screens moving straight or opening radially by a practically instantaneous objective (the front lens) and of any dieptrical system, such as that of compressed air. Changing the magnitude of the imulse it is possible to vary the velocity of the shutters, and in some cases an exposure of only of a second is reached.

Let us turn now to the problem of photographic objectives. As their basic properties were considered previously in subchapter 2, we shall merely touch upon their special characteristics which are significant in astrophotography.

In using the photographic objective it is necessary to know its light gathering power (or its photographic aperture) which is in practice determined by the ratio d/F, where d is the diameter of the objective and F is its principal focal length. Let us note that according to the well known rule of Gauss (see N. Zinger "A Course in Astronomy", Section: Practical Astronomy) it is always possible to find on the main axis of the dioptrical system of a complex objective two points which are called the main points and which are at equal distance (= F) from the main foci of the system and from which the object and its focal image subtend equal angles. This rule holds only within certain limiting conditions, namely, when the angle of inclination of the extreme ray falling on the objective to the central axis of the objective is sufficiently small (cos of the angle ~ 1). For the objectives of comparatively small field of vision as the astrophotographic objectives usually are, this condition is nearly fulfilled, but for the so called "wide-angled" objectives it is necessary to use a diaphragm which removes, more or less, the extreme rays and thereby lessens all those defects in the formation of the image of which we spoke in subchapter 2. In this case, "the diameter of the objective", or the diameter of "actinicaperture" equal to d is the diameter of the diaphragm when it is placed in front of the objective, and it is somewhat larger (determined empirically) when the diaphragm is placed inside of the objective -- between its main points ("iris" diaphragm). The optical constant of the objective, d and F, are usually approximately given by opticians, and the focal ratio $\frac{d}{F} = \frac{1}{p}$ is often denoted on the ring of the diaphragm, for its various sizes, in the form of F:p, but these numbers may be determined accurately by special experiments by photographing, for instance, a scale of a known length from various accurately measured distances.

The principal focal length F determines the scale of the photograph as well as the angular size of the field of view of the "photographic eye". "The utilized field of vision "or" the "field of sharp image" are used as technical terms, for, an account of residual fefects of the objective, the images gradually become worse as they recede from the center of the field. This fact is especially noticeable in photographing with long exposure the stars of the sky (the best test for any photographic objective). However the meaning of "sharp image" is of somewhat conditional character and it depends upon the requirements of the problem. Terrestrial photographs require a sharp image upon the entire plate (the objective "covers" the entire plate) and therefore only such sizes of plates are used the diagonals of which do not exceed the diameter of the field of sharp image; this rule is not applied to astrophotography and the plates used are of large size, but only the central part of the plate is utilized in measurements according to the requirements of the given problem.

It follows from what we have said that the use of the diaphragm will increase the field of sharp image, although its illumination will decrease. It is easy to see from geometrical constructions that the brightness of illumination on one square unit of surface of the plate (at the principal focus) will be propoetional to $(d/F)^2$.

It is necessary to mention another circumstance in connection with the construction of a simple photo-objective of two lenses. It is of great importance in photographic astrometry. If the spherical aberration etc. is well eliminated in the objective, the focal surface of the objective may be taken to be close to a spherical surface described from the rear principal point of the objective H (Fig. 8) by a radius equal to the principal focal length;

Let plate cD perpendicular to the main axis of the objective HF be tangent to this sphere at point F; this point is called the optical center of the plate, and the image of a star obtained at this point will have the form of a small circle blackened uniformly. On the contrary, the image (of a star) which is at a distance AF (in angular measure) from F will be circular only at a point A, but on the plate, at point B, it will have the form of an ellipse whose major axis is turned towards the optical canter of the plate. Evidently, measuring on the plate the distance BF we shall obtain the tg of arc AF, and not the angular distance AF of the two stars; this phenomenon is called the normal distortion of the objective and is taken into consideration in measurements. However, in practice we may ask the following question: Theoretically we should, in measuring set on the center of ellipse B, but will it be uniform so that we may easily find the center? In other words what will be the distribution of intensity within this ellipse? The answer to this question was given by the opticians Steingel and Abbe; the first of them investigated the objective of the Konigsberg heliometer and obtained results given in Fig. 9a and 9b. Here 9a is the front surface of the objective, on which 25 points are noted where 25 rays pass from an oblique pencil of parallal rays producing an image 50' from the center of the field of view. Fig. 9b shows the places where thecorresponding rays fall in the image and it shows the extreme one-sidedness and inequality of the distribution of light (similar to the phenomenon of coma). On the contrary, Fig. 9c represents (according to the computations of Abbe) the distribution of the same rays if in the construction of the objective the so called condition of sines were observed which may be briefly stated as follows: The middle and the edge of the objective must have the same true principal focus for rays of average refraction. Obviously, an approximation to this condition is very desirable for accurate astrophotographic work. We shall not describe the photo-objectives used in astromphotography in detail, we shall only say that they are divided into two fundamental classes: 1) objectives of short foci and of great light gathering power, almost always of may lenses, of small scale and large field of view; they are used mostly for pure astrophysical problems (photometry, spectral analysis), also for surveys ("Durchmusterung") and for the photography of faint objects (commets, nebulae; minor planets etc) with the purpose of an approximate determination of their position and 2) objectives of long focus, of comparatively small

light gathering power usually only of two lenses (rarely of three-lenses) having a small field of view, but a large scale and used for very accurate astrometric wo work (stellar parallaxes and proper motions, accurate positions of planets and their satellites, accurate positions of individual stars etc). Let us also mention the now widespread use of reflectors (mirrors) which on account of their large light gathering power and absolute achromatism are very useful for the astrophysical investigations of faint objects; they are less useful for pure astrometric work because of their small usable field of view and also for other reasons.

§ 26. The Chemical Part of Photography. Having obtained a correct image of photographed objects on the surface of the plehe, we must now consolidate it on the plate taking advantage of the fact that the light, or more accurately the radiant energy acts upon various bodies and chemical combinations making more or less profound changes in their physical and chemical properties. This change is, in general, an increasing function of the brightness of the light and of the length of its action as follows from Vant - Hoff's law: the quantity of the photochemically changed substance is directly proportional to the quantity of absorbed radiant energy. A corollary of this law is: only those rays which are absorbed act (Draper - Grotgus law).

Hence it is evident that theoretically many substances might have served for the purpose indicated above, but in practice only those are selected upon which light reacts strongly and which are easily consolidated. We have seen from the history of photography that such substances are mostly haloid salts of silver, i.e. silver chlorid ($AgCl$), silver bromide ($AgBr$) and silver iodide (AgI). The latter combination i.e. silver iodide was used in daguerreotyping in the wet collodion process which is of value even at present in astrophotography in photographying the Sun on account of its superior fineness of details obtained. Silver chloride is less sensitive than silver bromide (in the hidden image) and it is applied chiefly in the positive process (diaposition plates), but even its less sensitive plates (such as the so called Gaslight - plates, for instance) are also used for the photographing of the Sun and for very bright objects in general. Finally, the silver bormide is used in the preparation of less sensitive as well as highly sensitive plates of various kinds, and it is of the greatest importance in contemporary photography.

Upon the three chemical combinations indicated act, only the actinic rays i. e. the blue, violet and ultra-violet rays; but the maximum of their action lies in different parts of the spectrum for each of these chemical combinations (in the ultra-violet for $AgCl$, in the blue for $AgBr$, and about 438µµ for AgI). However, there are methods based on Draper-Grotgus law for making the bromogelatine plate sensitive also in other parts of the spectrum. Indeed, H. W. Vogel showed in 1873-75 that it is sufficient to "paint" the grain of the plate with some organic paints in order to make them absorb rays of definite length and thus the

Passing to the problem of determination of length of exposure for the obtaining of a hidden image and its so called development, we must consider a few of the hypotheses on the nature of these images. There are six hypotheses offered (three chemical and three physical ones), and more or less substantial objections may be raised against any of them, so that the matter is not definitely settled yet. But we shall take as a working hypothesis that as a result of the sufficiently strong action of the light, a certain amount of metallic silver is freed (in the latent image) the grains of which are centers of attraction for more silver that is liberated in the heloid. We should add the following; experience show that there is a so called "threshold" of sensitivity for each kind of emulsion as it is necessary to give it a certain minimum quantity of energy of light in order that the process of separation of silver metal may start.

Let us imagine that we are photographing a sufficiently bright star (or, in general, any bright point) and that we gradually lengthen the exposure; no sooner do we pass the threshold of sensitivity of the plate in the circular area where the image of the star is formed, few particles of silver bromide will suddently decompose and will liberate the metallic silver. Suppose this circular spot, which is called "the circle of limiting or limit image", contained although n particles of AgBr, and assume that in the first moment x particles were proportionality decomposed; it follows from Vant-Hoff's law, that the factor of quantity of silver separated should be proportional to the product i T where i is the brightness of the incident light (star's image) and T is the period of exposure (Rosco-Bunsen law). On the other hand we may assume that the further decomposition of AgBr will proceed more slowly as the number of limited image. Hence we may write:

$$dx = s\,(n-x)\,i\,dt$$

where dx is the increment of the silver sediment in the next element of time dt, and where s is some multiple of proportionality characterizing the sensitivity of the plate. Dividing the variables in the equation and integrating from 0 to x and from 0 to T, where T is the entire period of exposure, we have:

$$\int_{0}^{-x}\frac{dx}{n-x} = \int_{0}^{T} si\,dt = siT = -\ln(n-x) + \ln n$$

and hence:

$$\frac{n}{n-x} = e^{siT}$$

or taking the ratio $\frac{x}{n}$ as the rate of blackening we have

blackening $\frac{x}{n} = 1 - e^{-siT}$ (Mibhalke's formula)

From this formula the conclusion may be drawn that for long exposures the relation between the blackening of the negative and the exposure is best expressed by a logarithmic function; astrophotographic observations confirm this condlusion. However, we must state that Michalke's formula may have only an approximate value as it is derived on the assumption that the process of the formation of the hidden image is a purely photochemical process. This would be so if the gelatine itself were of no influence in the process. But experience and observations show that the gelatine is not indifferent to this process; it may, for instance, hold back and even collect the separating haloid which acts on the silver and thus the process may slow down, stop or even go into reverse as it occasionally observed (the phenomenon of solarization). Schwarzschild's investigations and those of other show also considerable deviations from the Rosco-Bunzen law: instead of the expression: iT = const the following formula is offered

$$I T^{p} = const.,$$

where p = 0.8 - 0.9.

For terrestrial photographs were T (or iT) is a small quantity, Michalke's formula may be transformed into a series, and limiting ourselves to the first order, we have:

blackening = 1 - (1 - s iT+ ....) = siT = const.

Hence, the length of the exposure is $T = \frac{c}{si}$,

where i is the brightness of the image on the plate. It depends on the brightness of the outside illumination I and on the optical constants of the photo-objective F and d, namely:

$$i = AI\frac{d^2}{F^2}$$

and therefore:

exposure = $T = B\frac{F^2}{sId^2}$

(B is the factor of proportionality)

Hence the quantities s, d, and F are known but the brightness of the outside illumination I is determined with difficulty, for it depends on the position of the Sun above the horizon, on the weather, on the transparency of the air and on

numerous other causes. Therefore a séries of instruments (photometers, actinometers) and tables are available by which all the indicated factors may be computed and the order of the exposure determined even it but approximately. Of these instruments let us mention Winn's actinometer, in which the brightness I is determined by the time of blackening of the attached positive paper, and the ingenuously constructed tables by Mitkevich and Golubovsky (USSR) and which appeared in several editions (the so called "Mitgol" tables).

Before we go into the problem of the development of the latent image we shall explain how the expressions "correct exposure" and "the correct or normal negative" are to be understood. We may give the following purely practical definition: a normal negative -- in the sense of accuracy of the exposure is one which yields to positive corresponding accurately to the form of the photographed object in the sense of distribution of light and shadow. Usually three gradations in the illumination of the photographed object (or panorama) are distinguished: that of high lights, i.e. the most illuminated places; that of shadow -- the least illuminated places, and that of half-tone-- the middle-ground between the first two gradations. If the exposure is so small that the sediment of silver is obtained only in the gradation of "high lights" and "half-tone", details in the gradation of "shadow" are absent, or there is no sediment of silver at all. Such a negative is then called a under-exposed and a too contrasted negative. On the contrary, if the exposure is more than normal it may happen that the gradation of "high lights" oreven that of "half-tone" may begin already to solarize, i.e. to become gray instead of becoming black approaching gradualy the gradiation of "shadow". The result is an over-exposed or a very flat negative.

All these circumstances become clearer in the investigation of the so called characteristic curve of a plate of a given sort. This curve shows graphically the relation between the blackening and the length of exposure for a given sorce of light. (see ch. III, 14).

Having obtained the drvelopment of the latent image it is necessary to render it visalbe i.e. to develop the exposed plate. The essence of this process is that, in addition to the minimum sediment of silver coused by light in the latent image, a new, considerably large amount of silver is added either from the developer or from the reserve of AgBr which is still contained in the layer around the latent image (a chemical reaction). In this process the grains of silver in the latent image become larger and visible similarly to the way they increase in the daguerreotype because of the sidiment of particles of mercury.

The best restorer of metallic silver out of its haloid salt is hydrogen at the moment of its isolation, and we can obtain the hydrogen by decomposing water by means of some substance which becomes greatly oxidized in the presence of

insolated silver (according to the latest theories the insolated silver is a catalyst here); such a substance is called the radical of the developer. Thus the reaction proceeds first according to the following equation:

$$2AgBr + H_2O - O = 2HBr + 2Ag \ldots$$

(the oxygen is taken away by the radical).

As a haloid acid is formed, it is necessary to neutralize it, otherwise the reaction may stop or even turn backwards. Therefore, it is necessary to introduce into the developer some alkaline substance; such substances are caustic potassium, caustic sodium, ammoniac, soda and potash. Finally, if the developer is prepared in the form of a single liquid in concentrated form, it is necessary to add to it what serves as a preservative which is sodium sulfate ($Na_2SO_3 + 7\,H_2\,O$); its role is to fix upon itself the oxygen of the air which might have oxidised the radical with the preservative and 2) the solution of the alkaline substance, and then they are mixed in a definite proportion of the so called "direction of development".

There are numerous alkaline developers on the market for sale. This is explained by the fact, as the bros. Lumiere have shown, that a series of organic, so called "aromatic" combinations (derived from benzol - C_6H_6) may be used under suitable conditions as radicals of an alkaline developer, and most of the developers are distinguished accordingly. Of those mostly used at present we shall name: pyrogallol (used already by Talbot), glycine, hydroquinone, methol and rodionol.

Besides the alkaline developers another is used. It is the iron or, as sometimes called, the acid developer; the radical in it is the oxalic iron oxide (FeC_2O), and no special alkaline substance is introduced because it appears in the reaction of development. This developer is less energetic than the alkaline developers, but it has several special merits: 1) It is always fresh and is of constant composition as it is prepared just before the developmentof the solutions which are preserved (iron sulphate - $FeSO_4$ and oxalic potassium oxide - $K_2\,C_2\,O_4$), 2) It is cheap and it does not require any imports, 3) its action at a definite temperature (+15°C) is always uniform and therefore it may be used as a developer for hours which fact is important in astrophotography. (It is impossible or very difficult to watch over the development of celestial photographs by the human eye). In general, the iron developer may be recommended for photographic astrometry and even for photographic photometry (It is free of the so called "Eberhardt phenomenon"). On the contrary, when very faint objects are evoked on the negative and also in expeditions, it is necessary to use an energetic and very concentrated alkaline developer as is radinol, for instance.

The operation of development as other operations with photoplates (fixation, sensitization, the change of plateholders etc) is made in a special dark room and in a non-actinic (red) light. In the case of orthochromatic or panchromatic plates a special illumination is used (with the aid of light filters) or it is sometimes necessary to work in complete darkness. The development of a normally exposed plate takes usually about 7 - 8 minutes, but in case of under exposure or over-exposure it is necessary to direct the development by change in the composition of the developer (alkaline), or adding to it drops of a retarder or of an accellerator. As a retarder potassium bromide (K-Br) may be used the addition of which retards the development, but it greatly increases the contrast of the negative so that it is recommended for use in over-exposures; very under-exposed plates and negatives are very difficult to correct by any subsequent operations. If it is possible to investigate the photographs in regard to their transparency (in red light) it is best then to judge of the accuracy of the exposure by the details of the "shadow" gradation, and it is best to judge of the accuracy of the development by the density of the "high light" gradation.

Having developed a plate it is now necessary to fix it (before exposing it to light) i.e. to remove from the layer all remaining silver bromide which was not decomposed either by light or by the developer; this op ration is carried out in a solution of hyposulphite ($Na_2S_2O_3$) which transforms the silver haloid into a soluble compound. Fixation lasts for about 15-20 minutes until the plate becomes entirely transparent; then the plate is washed in running water in order to remove from the gelatine any traces of hyposulphite and other salts. Astronomic negatives which are to be preserved for many decades should be carefully and thoroughly washed (for many hours) and it is also recommended to subject the plates to alumination, i.e. to treat them with a saturated solution of potassium alum (K_2SO_4, $Al_2\ (SO_4)_3 + 24\ H_2O$) for 1-2 minutes in order to harden the gelatine layer and to make it last longer.

Further operations on negatives when they are still unsatisfactory consist in weakening or strengthening them. The essence of the first operation consists in the substitution of the silver sediment tracing the image on the negative by a more opaque chemical combination. The most frequent method of strengthening used in the application of mercuric bichloride (H_gCl_2) or of uranium salts. The weakening of the negative i.e. the removal of parts of the silver sediment is made by means of a solution of the salt $Fe_2k_6\ (CN)_{12}$ in hyposulphite which transfers the metallic silver into a soluble combination. After all these operations the negatives should be well dried without any access of dust and be preserved in a dry place in order to avoid any development of bacteria on the gelatine, for that would be disastrous for astronegatives.

Because of lack of space we cannot enter here into a more detailed description of various photographic operations or precepts for which we refer the readers to numerous special handbooks. We do not attempt to describe here the

positive process i.e. on the printing of negatives (in the original or enlargement) on diapositive plates or on various kinds of paper sensitive to light. For this process as well as the photo-mechanical processes serve chiefly for the illustration of scientific and other publications and for the production of any amount of copies of an original photograph. As material for scientific research original negatives should be used where ever possible, and for accurate measurements (in astrometric and photometric supplements) the use of original negatives is indispensable.

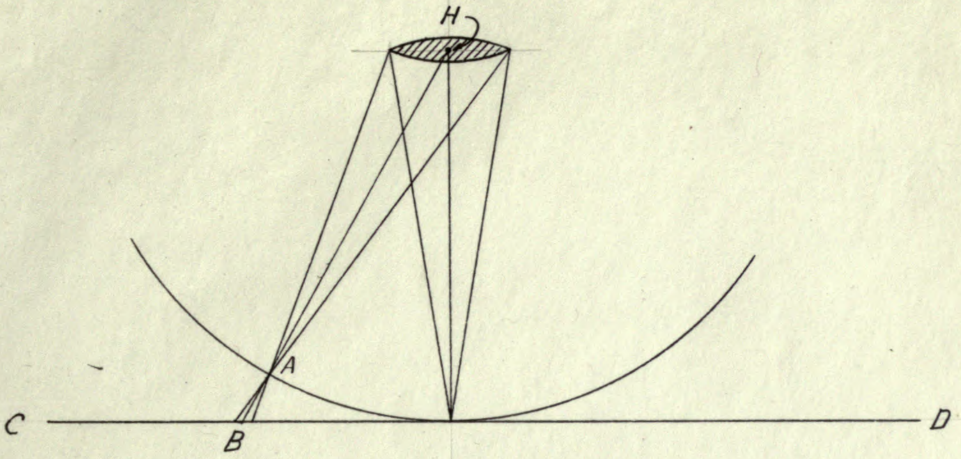
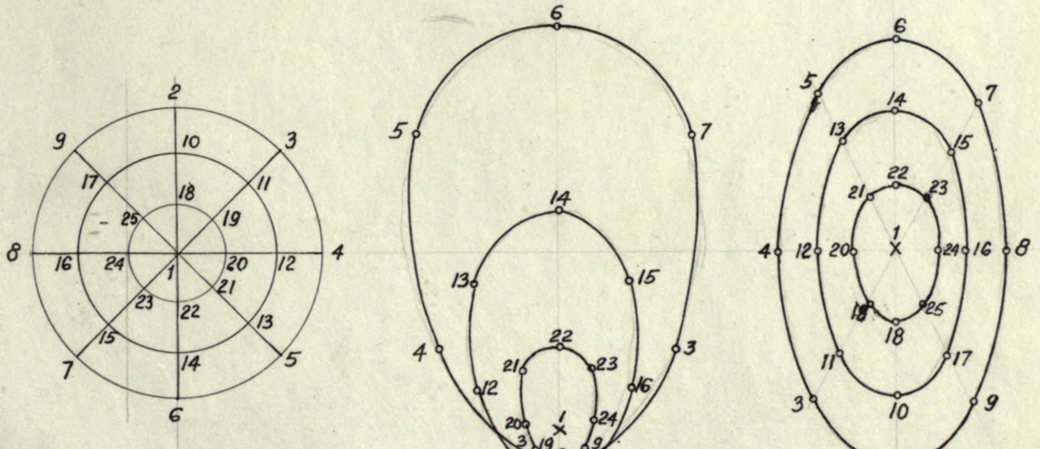


Fig. 8.



a. b. c.

Fig. 9.

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Chapter I.

Foundations of Photographic Astrometry.

by S. K. Kostinsky

1. Determination of Coordinates of Celestial Objects.

§1. Astrophotographic Observations. Astrophotography -- a method consisting in the substitution for the human eye (aided as well as unaided) of the "photographic eye" -- is equally applicable in astrometry, where the dierection of the rays of light is studied, as well as in astrophysics where the composition of the rays is investigated. Here we shall treat only the application of photography to purely astrometric observations the purpose of which is the accurate determination of positions of celestial objects and the changes of these positions under the influence of parallaxes, proper motions, etc.

The instrument for astrophotographic observations is the combination of the photocamera with the visual astronomical telescope mounted parallactically, or the so called astrograph (photographic refractor). Depending on the problem at hand the structure of astrographs may vary, but essentially it is determined by the type and optical constants of the photo-objective with which the astrograph is equipped. We have previously said that from this point of view the astrographs are divided into two groups: those of long focus (of low light gathering power) and those of short focus (of high light gathering power). As an example of the first group in the USSR the large astrograph of the Moscow Observatory (d = 38cm, F about 6m) and the two normal astrographs in Pulkovo and Tashkent (d = 33 cm, F = 3.4m) may be mentioned; of the second group the Bredichin Astrograph at Pulkovo (d = 17 cm, F = 80 cm) and the equatorial camera in Moscow (d = 11 cm, F = 64) are examples; besides, there are a few astrographs of short focus in Pulkovo, Simeis, Kazan, Kharkov etc.

Next we shall speak of the expreience obtained in the work with the Pulkovo normal astrograph which is the more typical and the more universal instrument for astrometric work (Fig. 10), but we shall first say a few words about the selection of different types of astrographs for various purposes.

a) For photographs of faint, elongated objects (faint nebulae, comets, faint minor planets etc) it is better to use astrographs of short focus having complex, wide-angled objectives of average size (for instance, d = 15 cm, F = 60 cm) or reflectors, wherever a small field of view is sufficient for the purpose; for photographs of the Milky Way, zodiacal light, for survey (Durchmusterung), of variable stars, etc special objectives of very short focus (F: 2, f: 3) and even simple lenses are used.

b) For stars it is not necessary to have a very large aperture as compared with the focal distance; the best focal ratio is about f:10 (for instance, d from 30 to 40cm, F from 3 to 4m); for brighter stars it is necessary to decrease the aperture to 15 - 20cm; on the contrary, for photographs of close double stars, and also for the determination of stellar parallaxes it is necessary to use astrographs of longer focus.

c) For detailed photographs of bright surfaces (Sun, Moon, major planets) astrographs of very long focus and of small aperture should be used for a magnifying system (a photoheliograph, for instance) may be used instead.

The Pulkovo astrograph of long focus is called "normal" because by its optical constants and by some of its other detailed it belongs to that type of astrographs which was developed in the eighties of the last century for the international undertaking in photographing the entire sky. It consists of a single metallic tube of oval cross-section, but separate inside the tube along its length by a opaque partition; on the upper end of the tube are fastened two objectives of two lenses each: one is a photographic objective of 33cm aperture and of a focal distance of 3.46m, the other is a visual objective (d = 25cm, F = 3.6m); on the lower end are the movable part on which a metallic plate holder is mounted and the eye piece for the visual tube which may be moved in declination and in hour angle by micrometric screws to about 30' on either side of the optical axis. This contrivance is called the "photographic micrometers" and it serves for setting on the so called "guide" star if it is located off of the optical axis. Inside the photographic tube, in front of the plate holder, a shutter is placed which opens and closes by means of a pot-rack and pinion mounted outside. During observations a low powered eye piece (magnifies 40 times, field of view 30') is used for the search of the guide star, and a high power eye piece which magnifies about 380 times is used for "guiding" on the star.

The parallactic mounting of the Pulkovo astrograph, as may be seen from Fig. 10, is between that of the German and that of the English systems, as here the upper part of the column is parallel to the Earth's axis, which fact is of great convenience for long exposures; the clock mechanism moving the tube is usually of "Repsold" construction with a conic pendulum which makes it possible to regulate its motion by means of weights at the end of the pendulum (in latest constructions, as for instance in the Pulkovo zone astrograph, for the regulation of the clock mechanism is done by an electric contrivance with the aid of second signals from the normal clock of the observatory); this part of the construction of astrographs is always subject to attentive, vigorous scrutiny: the cogged wheel of the clock mechanism and the worm transmission must be entirely free of periodic errors and well fitted to each other, otherwise it will show upon the image of a star on the plate in that it will appear systematically elongated.

The photographic objective of the astrogrpah after being tested by Hartmann's

method appeared to be perfect in absence of spherical aberration and of astigmatism, but it is achromatic only in part, namely; chiefly in the focus the rays of wavelengths of 397 μμ and 434 μμ are concentrated a square plate of 16x16 cm is used on which the free field is 13x13 cm; this gives a field of vision for the astrograph of 2° 10' x 2° 10', as for all astrographs of normal type the angular scale is nearly equal to 1' per 1mm; the field of view which is utilized for accurate observations is, do to be sure, somewhat smaller (from two to three square degrees).

Before the sky is photographed by the astrograph it is necessary to check the mounting of the astrograph, to verify the motion of the clock mechanism, and to determine the position of the main focus of the photographic objective as well as of the center of the plate. The checking of the mounting and the determination of the remaining error (i.e. the non parallism of the polar axis of the instrument with the axis of the earth) is done, usually for refractors, by means of visual observations of stars near the meridian and of stars near the pole at elongation; this may be done also by photographic traces of similarly selected stars on the same plate; the motion of the clock mechanism is verified before the beginning of the observations by some equatorial star. The position of the main focus and its dependence on the temperature is investigated specially by means of extra focal photographs; also by means of a special investigation is determined the position of the optical center, i.e. of the base of the perpendicular dropped upon the plane of the plate from the rear Gauss point, the main point on the objective (K. Olsson's method — by means of a very small diaphragm, placed in front of the center of the objective, and of a mirror plate in which the pupil of the observer's eye is reflected); to be sure, the optical axes of the visual and of the photographic tubes should be parallel to each other and the frame is so regulated that the optical center is nearly coincident with the geometric center of the plate.

Besides, in observations by astrographs it is necessary to know their so called "penetrative" power; by this we maen the relation between the length of the exposure and the magnitude of the faintest star which may appear on a negative; evidently, this relation depends upon the optical constants of the objective, also upon the atmospheric conditions (transparency) upon the sensitivity of the plates used, and upon the selected photometric system of stellar magnitudes; by photographic well known standard regions of the sky of stars of various brightness photometrically well determined, it is possible to derive an empirical formula for each instrument under ordinary conditions. As an example and as a means of comparison we give the following two formulae for two Pulkovo astrographs, one of the normal type, the other of the Bredichin short focus type:

Normal astrograph of long focus:

Brodichin astrograph of short focus:

T^s is the length of exposure in seconds, m is the magnitude of the s'
'onner Durchmusterung system) on the plate.

It follows from a comparison of these formulae: 1) that the normal astrograph gets stars of the 10the magnitude about three times as fast as the Bredichin astrograph; this corresponds to the theory, as the area of the objective of the former is about three times as large as that of the latter, and 2) that the normal astrograph, as the exposure increases 2.5 times, gains only 0.8 of stellar magnitude while the Bredichin astrograph gains more, which fact may be explained by a better achromatization of the latter's objective (Petzval's system) and by other causes (difference in focal length absorption of light in objective etc) which are difficult to compute theoretically.

The process of photographing by an astrograph consists, in general, in the following procedure: having checked the accurate performance of the instrument and having established the photographic focus (as a function of the temperature), a filled plate holder is put on and the place in the sky of the guide star is looked for by means of divided circles (as usually is done on refractors). Setting the clock mechanism in motion, the guide star is set on the cross hairs in the eye piece of the guiding tube and them the shutters are opened, first those of the plate holder and next those in the tube noticing mean while the time of the beginning of the exposure by the chronometer or clock. Beginning with this moment the astronomer must "hold" the guide star on the cross hairs during the entire exposure, and if the star gets off the cross hairs for any reason it is necessary to return it back by using the slow motion of the instrument in the two coordinates. In photographing by instruments of long focus the so called Ritchey plate holder is used which is connected with the eye piece and which is movable by micrometer screws independently of the entire instrument. The purpose of such "holding" or "guiding" is that the images of the stars on the plate shall always remain on the same places during the entire exposure; otherwise the stars will appear on the negative not round but elongated in one or the other direction which fact would render the measurements of the plate difficult and lead to systematic errors. The chief possible causes for the elongation of images are as follows: 1) residual errors in the motion of the clock mechanism, 2) a residual error in the mounting of the astrograph, i.e. the non coincidence of the pole of the instrument with the pole of the sky, and 2) the influence of differential refraction. The first cause which is easily removable (together with the error of "guiding") acts in the meridian as well as off it whereas, the other two causes show their influence most at large hour angles which increase as the exposure increases and as the zenith distance of the photograph increases; they cannot be removed by "guiding" because there by the stars on the plate describe arcs of a circle whose center is the guide star. Hence the so called "hour angle errors" (according to Kapteyn) appear in accurate measurements of such plates.

In conclusion let us say a few words about the influence of atmospheric conditions upon the astrographic observations; the wind has a harmful influence (especially on the instruments of large size or on those which are unstable) causing the elongation of images and even creating sometimes unreal satellites for bright stars; on the other hand, the influence of bad "conditions of images" (washed out, unstable images) show itself less on photographic plates than on visual observations as it mostly widens the round images of stars on the negatives.

2. On Measurements of Astrograms (Photographic Plates). In order to measure the relative position of stars and other objects on astrographic plates (astrograms) various methods are employed, depending on the problem and on the required accuracy beginning with the application of a simple millimetric ruler (or net) and ending with very complex measuring instruments. At present there are numerous types of measuring devices for astrograms, but the bases of all of them are the accurately subdivided scale and the micrometric screws (sometimes only one long screw). We shall describe here only two types of instruments which are used most in the researches of stellar astronomy.

a) The Universal Measuring Apparatus of the System Devised by D. Gill and Van-de-Sande Backhuysen (Figure 11). This instrument which was proposed in 1892 for the measurements of photographs of the international photographic catalogue of the sky was constructed in large numbers by I. Repsold. It consists of a round metallic base which may be placed obliquely if necessary for the convenience of measurement; to the base, on a separate arc, are fastened three microscopes with micrometers (and a magnifying glass on the side) of which the middle micrometer can move upon a steel rail parallel to an accurately subdivided milimetric scale. By means of a special level this microscope may rotate on the plane, perpendicular to the subdivided scale, and facing in turn the scale and the plate measured; the latter is set into a special round frame (with a square opening) which moves by means of a rack on the leading cylinder, perpendicularly to the rail with the moveable microscope and, hence, also to the subdivided scale. The moveable frame is equipped with a subdivided circle (10') and by means of two unmovable microscopes the position angle may be calculated within the accuracy of 1"; besides, the position of this frame on the leading cylinder may be noted through the magnifying glass on the side scale within the accuracy of 1.1mm. The movable microscope magnifies usually about 12-15 times (a larger magnification of 25-30 times is seldom used) and one rotation of the screw is about 0.5mm; the least reading given on the recording drum is within 0.001 part of a rotation.

Each measuring device should be investigated in detail in regard to relative michanical errors of its construction. For the type of instrument under consideration the following possible errors may be mentioned: 1) errors in the subdivision of the main scale, 2) curvature of the leading cylinder, 3) periodic and progressive errors of micrometric screws, and 4) errors of projection, i.e. the plane of rotation of the microscope is not quite parallel in various places in its motion on the rail. It is necessary to note that these errors in the Repsold instruments are very slight, and tables are easily made for an individual instrument to correct the measurements for all technical errors (see F. Renz and S. Kostinsky. "Untersuchungen des Repsold'schen Messapparates fur photographische Sternaufnahmen". Publications of the Academy of Sciences, 1896, Russia).

Besides the technical errors of the instruments it is also necessary in the measurement of plates to take into consideration accidental as well as systematic errors of the setting of the cross hairs of the micrometer on the images of stars; the former, the accidental errors, are not dangerous as they

may be considerably weakened by increasing the number of measurements (or the number of plates for the same part of the sky); however, there is a systematic personal error of measurement consisting in the inaccurate estimation of the position of the center of the image of the star; this error is an increasing function of the diameter of the image, and it is partly overcome by turning the plate by 180° or by turning the image by means of a reversing prism and ta taking the mean of the measurements in the two opposite positions. Next, a systematic error may depend on the incorrect form of the image distorted by causes indicated above or because the image is too close to the dege of the plate; these errors may be overcome by the increase of the number of plates and by varying the conditions of their exposure. Finally, let us mention the posibility of deformation or the so called "distortion" of the gelatine layer depending on the chemical development of the plate. Fortunately, numerous investigations showed that the distortion of the layer is, in general, very small and is of accidental character, except the edges of the plate where no measurements are made and where the distortion may be large. Cases of a purely mechanical injury to the layer (such as scratches etc) are rare, but when they occur they may entirely spoil the plate.

With the Repsold instrument described here it is possible to make measurements in rectangular as well as polar coordinates of images of stars and of other objects obtained on the plate. Usually, the rectangular coordinates are measured, and the optical center of the plate is taken as the origin of the coordinates. The plate itself is so placed on the instrument that the projection of the celestial meridian on the plate is nearly parallel to the motion of the leading cylinder (X axis and it is therefore nearly perpendicular to the main scale X axis). This orientation is made by means of two stars whose positions (equatorial coordinates) are known. In practice the measurements consist in the alternate setting of the micrometer cross-hairs of the moving microscope on the image of the star on the plate and on the nearest line of the main subdivided scale as if projecting the image of all stars on the scale. Having obtained for each star (or some other object) the corresponding reading on the scale (corrected for all errors) and comparing it with the computations with respect to the optical center we shall obtain the value of the X coordinate (usually within 0.0001mm). In order to obtain y the moveble frame with the plate is turned 90° by means of the position circle, and then a similar measurement is made again. An approximate computation with respect to the optical center is obtained from the computations with respect to the leading star whose position with respect to the optical center is determined on the astrograph by means of a photographic micrometer.

Besides the universal described device, Repsold constructed a somewhat simpler instrument specially built for quick measurements of photographs for the photographic catalogue of the sky. The accuracy subdivided scale is absent in it, and the frame with the plate moves by mutually perpendicular guide cylinders; the microscope is immovable and is equipped with a double micrometer by means of which measurements are made simultaneously in both coordinates. For this instrument is is necessary to print the so called

Gothier grid on each plate. This grid consists of fine lines spaced every 5 mm. The measurements of the stars are made relative to these lines, whose p positions (the errors of the divisions of the master net) are regarded as known. This apparatus produces quicker, but not as dependable results, due to the errors of copying the grid, the super imposition of lines over the images of stars, etc.

b) The Zeiss Stereocomparator - (Fig. 12). The Repsold apparatus and other contrivances similar in type, produce the absolute values of the rectangular coordinates of the stars, and these are later transformed into the equatorial coordinates by means of computations. However, when it is necessary to find only the differential variations of one or other kinds of coordinates, (which may depend on the parallax, proper motions of stars, or other causes), it is much quicker and more accurate to measure these differential variations directly, by means of a comparison of two plates, and to limit the determinations of positions of the object to the approximate measurmeasurements.

For this particular operation, C. Pulfrich of the Zeiss organization, has designed the apparatus, known as the stereocomparator, or simply, comparator.

The Zeiss comparator consists of a substantial metal stand, equipped with an oblong vertical frame, which can be moved by means of special controls in horizontal and vertical planes. The frame runs on rails. The displacements of the frame can be measured on suitable scales divided to 0.05mm. The frame has two openings illuminated from the rear by means of mirrors, and both openings are fitted with special holders for the negatives, which are being compared.

The plate holder on left can be moved roughly in both coordinates and can also be rotated in the vertical plane by means of a micrometric adjustment.

The plate holder on right, conversely, has rough adjustment for rotation, which is read on a graduated limb, and fine adjustments in the directions of the coordinates, which are read by means of micrometers and magnifiers, to 0.01 mm.

The apparatus is equipped with one of the following optical divices: either 1) the Helmholz "Stereomicroscope, wit an enlarged distance between the orulars, or 2) the "blinkmicroscope" (see Fig. 12).

The object of these optical devices is to superimpose the images of two plates, produced by two oculars of the stereomicroscope, or to make them appear simultaneously in the single ocular of the blinkmicroscope. This is achieved by an arrangement of several prisms of total reflection and of the objectives which enlarge the images 6-8 times; in addition to this, the blinkmicfoscopic has also a revolving screen, which allows to view the two images alternately (the "blinking"): a special mechanical arragngement allows to perform this "blinking" very rapidly.

The pair of plates in the comparator are oriented in such a way, that a stereoscopic image is formed in the stereomicroscope; then, if during the interim between the two exposures all objects on the plates retained their positions, then all the images will appear to be on the same plane. On the contrary, any star (or other object) which has changed its position will appear suspended either in front or behind the general stereoscopic plane, and this advance (or recision) will be proportional to the actual displacement of the images on two plates in direction parallel to the stereoscopic base.

An accurate measurement of this displacement, known as the "stereoscopic parallax", is done by means of movement of the right plate, relative to the left, or by means of a special micrometer screw, attached to the right ocular of the stereomicroscipe.

In the case of the blink microscope the plates are so oriented that during the rapid blinking all stars appear stationary (the images of both plates coinside exactly) the effect of a shift of any of them will be expressed by its swift jumping in the direction of its true motion on the sky. In order to measure accurately such a shift a filter micrometer with a position circle and two micrometric screws are attached to the blink microscope.

The described instrument is of important application in the direction of noticeable proper motions of stars, variable stars etc; measurements made with it are of great accuracy; for instance, according to investigations in Pulkovo, the annual proper motion of a star in an interval of 10-12 years between the photographs is determined with an accuracy of + 0".004 to + 0".006. However, it needs be said that this instrument on account of its large size is more subject to influences of temperature than the Repsold measuring apparatus; this fact diminishes somewhat its merits.

§ 3. On the Reduction of Astrophotographic Measurements. We shall deal in this section with the most widespread methods of reduction of the measurements on the plates from rectangular coordinates into equatorial i.e. into right ascension, α, and declination, δ. As the astrograms represent a central projection of a part of the celestial sphere upon the plane of the plate and as the center of a projection is the rear main point of the objective, then, describing from this point a sphere of radius equal to the main focal distance of the astrograph, we may imagine the plane of the plate as tangent plane to the celestial sphere and the point of tangency coincides with the optical center of the plate. From the theory of the central projection it is known, as may be easily seen geometrically, that the celestrial meridians (and, in general, the great circles of a sphere) are projected on the plane of the plate by a pencil of straight lines coverging in the projection of the celestrial pole on the same plane. The celestrial parallels (small circles) are projected on the same plane by a pencil of conic sections, i.e. in the general case, by a hyperbold (closer to the equator) and by an ellipse (closer to the pole). (This projection is identical with the so called "Lorenzoni's net or projection" or "zenith projection" which is used for the tracing of meteors).

We have previously said that the rectangular coordinates x and y of a star measured on the plate are usually related to the axes directed along the projection of the celestial meridian and perpendicular to it, and to the origin of coordinates nearly coinciding with the optical center of the plate. These coordinates x and y are apparent, i.e. influenced by differential refraction, aberration etc. and they are expressed in linear measure (in millimeters). Let us now assume that we had corrected the measured coordinates for the indicated effects, and then turn axis y through an angle so that it will exactly pass through the projection of the celestial pole on the plate (the visible or mean position of the pole for some epoch), next transferred the origin of coordinates

into the optical center, and, finally, transformed the coordinates x and y into an angular (or arc) measure by the known scale S; then we may write the following equation:

$$X = s\cdot(x\cdot\cos\varphi - y\cdot\sin\varphi + z);$$

$$Y = s\cdot(x\cdot\sin\varphi + y\cdot\cos\varphi + z'); \qquad (1)$$

The transformed coordinates X and Y which are referred exactly to the celestial axes are called ""ideal" or fundamental (also standard) coordinates, and they are evidently, functions of the equatorial coordinates of a given star and of the projection upon the celestial sphere of the optical center of the plate Let us note that the scale S is connected with the main focal length F of the astrograph by the formula

$$s'' = \frac{1}{F\cdot\sin 1''},$$

where F is expressed in millimeters and S" is the number of angular seconds in 1mm; besides, it is necessary to keep in mind that the influence of precession and nutation, which does not change the mutual positions of the stars, is taken into consideration by the previously given linear transformation of coordinates.

Let us now derive the formulae, connecting the ideal coordinates X and Y with the equatorial ones α and δ for a star, and A and D for the optical center of the plate. Let plate AB (Fig. 13) be tangent to the celestial sphere at point O' (optical center) and let O be the center of projection, i.e. the center of the celestial sphere the radius of which OO' = F we take as unity. We project stars S, which is on the sphere (in front of the drawing), from the center of the sphere on the plate at point S' and let us draw the celestial meridian PO' (circle of declination) through pole P and the optical center O'; the projection of this meridian on the plate O'Y will be our ordinate axis, and, if we draw S'Q' perpendicular of O'Y, then

$$S'Q' = X \quad \text{and} \quad O'Q' = Y.$$

Next, let us draw a great circle SQR of the sphere through the star and S perpendicularly to PO' and let point Q be the point of intersection. It is evident from the symmetry of the drawing that Q' is the projection of point Q on the plate (S'Q' is the projection of the great circle SQR). Finally, we draw in the plane triangle S'OQ' the line S"Q parallel to S'Q'. Denoting the polar distance of the points Q, S and O' by q, p and P and noticing that the

angle SPO' (at the pole) is equal to the difference of that right-ascensions of the star S and of the optical center, i.e. it is equal to $\alpha - A$, we obtain the following from the right angled spherical triangle SPQ:

$$tg\, SQ = \sin q \cdot tg(\alpha - A) \quad \text{and} \quad tg\, q = tg\, p \cdot \cos(\alpha - A), \qquad (2)$$

but tg Sq equal to the section S"Q (as OQ = 1) and from the similarity of the triangles OS"Q we have:

$$\frac{S''Q}{S'Q'} = \frac{OQ}{OQ'} = \frac{1}{\sec O'Q},$$

where arc O'Q = P - q; hence

$$S''Q = S'Q' \cdot \cos(P - q)$$

and hence

$$S'Q' = X = \frac{tg(\alpha - A) \cdot \sin q}{\cos(P - q)}. \qquad (3)$$

Finally, it is evident from the drawing that O'Q = tg of arc O'Q or:

$$Y = tg\,(P - q) \qquad (4)$$

The formulae (2), (3), and (4) are called Turner's formulae and they solve the problem. If we transfer from polar coordinates into equatorial ones i.e. if we substitute $p = 90° - \delta$, $P = 90° - D$ and if we eliminate the auxiliary angle q, we obtain the following relations in reversed form:

$$tg(\alpha - A) = \frac{X}{\cos D - Y \sin D} \quad \text{and}$$

$$tg\,\delta = \frac{\sin D + Y \cdot \cos D}{\cos D - Y \cdot \sin D} \cdot \cos(\alpha - A). \qquad (5)$$

It is evident from equality (1) that in order to "tie" our plate to the sky we must determine four quantities, namely: the scale S, angle of rotation (the so called angle of orientation) and corrections Z on Z' to the originally accepted coordinates. Knowing, approximately, the equatorial coordinates of the optical center A and D and having selected two stars (of those measured on the plate) α and δ of which are known, we compute for them the ideal coordinates X, Y according to Turner's formulae and by the known quantities from measurement x and y (corrected for refraction etc) we obtain four equations for the determination of the four quantities mentioned above, which are constant for the entire plate. Obtaining those "constants" we may compute conversely X and Y by formulae (1) and then compute α and δ by formulae (5) for any object measured on the plate and whose position is to be determined.

This method is called "the method of four constants". Its defects are: 1) the necessity of separate computations of refraction by quite complex formulae, 2) the combination into one system of equations those with respect to α and those

with respect to δ and 3) the assumption that the scale is the same along the x ad and the y axes and that the axes are strictly orthogeonal (which fact may not be true in reality). But Turner offered another method, "namely": the method of six constants" which has more of the defects listed above, and is almost as accurate. The principle of this method is as follows: all reductions needed for the transformation of the measured rectangular coordinates into ideal coordinates (orientation, scale, influence of precession and nutation, aberration and refraction) are reduced almost exclusively to a linear transformation as a function of the same coordinates. It is necessary only to keep in mind (in exceptional cases) the terms of the second order of refraction (the terms of the second order of aberration are negligible); however computations show that the latter are very small and in case of necessity they may be computed separately; for instance: on plates photographed with the normal astrograph, for the edge of a plate (about 50' - 55' from the optical center) and for zenith distance = 45° the terms of the second order reach at maximum only about 0".05 - 0".06.

Thus in most cases by far we have the right to use the following form:

$$d.x + e.y + f = X \quad \text{and} \quad d'.x + e'.y + f' = Y. \qquad (6)$$

Here the coefficients d, e, f and d', e', f' are those six constants which are to be determined in order to tie the plate to the sky. Evidently, three comparison stars are needed which are situated on a straight line. Then all computations are carried on by the same formulae, but here the connection of the measured objects to the comparison stars is done separately along each coordinate which method corresponds to the chain method used in meridian observations.

In special very accurate determinations or in determinations of positions of numerous objects on a plate (for instance, for the photographic catalogue of the sky, for stellar clusters etc) three comparison stars are insufficient, and more comparison stars are taken which distributed more or less evenly upon the entire plate. Only such stars are selected the α and δ of which are accurately known from meridian observations or from some catalogue (especially of a xone catalogue of the Astron. Gesellachaft.(= A.G). It is inconvenient to take stars which are too bright, as the settings on their large photographic images are less accurate; usually stars of 8 - 9th magnitude are taken as comparison stars, and occasionally even fainter stars are used (from photographic catalogues). Let us note that the α and δ of basic stars taken are always the mean values for the equinox of the year of observation, but the proper motions of the stars (when they are known) up to the epoch of the time of photographing are taken into consideration. If the number of comparison stars is larger than three, a series of conditional equations of type (6) are set up which are solved by the method of Least Squares or otherwise.*

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*) For instance, see, S. K. Kostinsky. "A Graphic Method of Computations of Constants on Astrophotographic photographs" (Published on the Academy of Sciences, March 1916 (In Russian).

For astrographs of long focus having a small field of view the application of Turner's accurate formulae are not quite convenient in computation as small angles must be dealt with. In such cases it is advantageous to transform them into series and to neglect the unimportant terms of the high orders; we give their transformations made by the writer of this chapter for normal astrograph. As the angle $\alpha - A$ is usually small, it is evident from formula (2) that the arcs p and q are close to each other. Assuming $p - q = \varepsilon$ where ε is a small quantity, and substituting $p = 90° - \delta$ and $P = 90° - D$ we obtain:

$$q = p - \varepsilon = 90° - (\delta + \varepsilon), \text{ and } P - q = \delta - D + \varepsilon;$$

Substituting these values into the formulae (3) and (4) we obtain

$$X = tg(\alpha - A) . \cos(\delta + \varepsilon) . \sec(\delta - D + \varepsilon); \quad Y = tg(\delta - D + \varepsilon). \qquad (7)$$

In order to determine ε we substitute cos (α A) in formulae (2) by $1-2\sin^2 1/2\,(\alpha - A)$ then we have

$$\frac{tg p - tg q}{tg p} = 2\sin^2 \frac{1}{2}(\alpha - A) = \frac{\sin(p-q)}{\sin p . \cos q} = \frac{\sin\varepsilon}{\cos\delta . \sin(\delta+\varepsilon)}$$

whence, after simple transformation, we obtain an accurate formula for the determination of ε in the following form:

$$tg\,\varepsilon = \frac{2\sin^2\frac{1}{2}(\alpha - A)\sin\delta . \cos\delta}{1 - 2\sin^2\frac{1}{2}(\alpha - A) . \cos^2\delta} \qquad (8)$$

The second term in the denominator of this formula is a very small quantity even for high declination. Assuming it to be equal to zero and denoting the function $\frac{2 \times \sin^2 1/2\,(\alpha - A)}{\sin 1''}$ by K, and also substituting tg ε by $\varepsilon'' \times \sin 1''$

we have:

$$\varepsilon'' = \frac{1}{2} . K . \sin 2\delta \qquad (9)$$

Function K is found in many tables (in Albrecht's navigation tables, etc) and it serves for the reduction of circum - meridian altitudes. It is easy to see gemetrically (see Fig. 13) that function ε is the "curvature of the parallel" on the plate.

Because of the smallness of $\alpha - A$ and of $\delta - D$ (for a normal astrograph $< 65'$), formulae (7) may be simplified and the computation may be reduced to a series of tables. Indeed, limiting ourselves to quantities of the third order we may write the following for any of the basic stars (the number of stars as m):

$$X_m = (\alpha_m - A)^s . T . \cos(\delta_m + \varepsilon) . \sec(\delta_m - D + \varepsilon)$$

and

$$Y_m = (\delta_m - D + \varepsilon)' . \sin 1' + \frac{(\delta_m - D + \varepsilon)^3}{3}\sin^3 1',$$

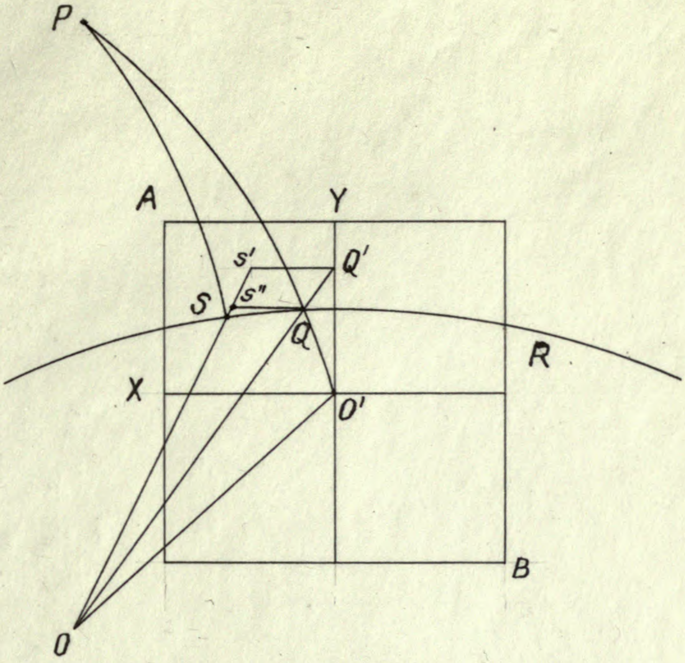
P
A
Y
s'
Q'
S
s''
Q
R
X
O'
B
O

Fig. 13.

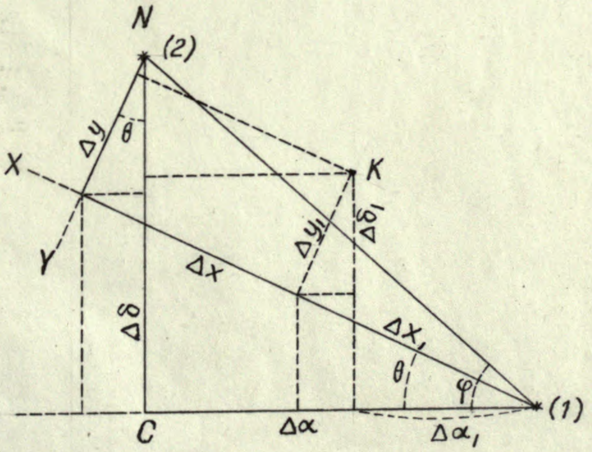
N
(2)
X
Δy
θ
K
Y
Δy_1
$\Delta \delta_1$
Δx
$\Delta \delta$
Δx_1
θ'
φ
(1)
C
$\Delta \alpha$
$\Delta \alpha_1$

Fig. 14.

where T = tgl^s and is taken from the table according to the argument $(\alpha - A)^s$ taking into consideration terms of the third order. The quantity Xm and Ym are expressed here in radians but it is more advantageous to express them in minutes of arc (as, for the normal astrograph, 1mm = 1'), we finally obtain:

$$\left.\begin{aligned}\frac{X_m}{\sin 1'} &= (\alpha_m - A)^s \cdot \frac{T}{\sin 1'} \cdot \cos(\delta_m + \epsilon) \cdot \sec(\delta_m - D + \epsilon) \\ \frac{Y_m}{\sin 1'} &= (\delta - D + \epsilon)' + \frac{(\delta_m - D + \epsilon)^3}{3} \cdot \sin^2 1'\end{aligned}\right\} \quad (10)$$

the term of the third order in the formula for $\frac{Y_m}{\sin 1'}$ represents the so called "normal distortion" (see introduction § 25): this term as well as the slightly changing functions $\frac{T}{\sin 1'}$ and sec $(\delta_m - D + \epsilon)$ are easily transformed into tables; in order to compute $X_m : \sin 1''$ we must use six - place logarithms as the measured coordinates with which the ideal coordinates are compared often have six figures.

Comparing formulae (6) written on the basis of Turner's principles with formulae (1) we notice that the coefficients d and e' express chiefly the influence of scale $(\cos\varphi \sim 1)$, and that the coefficients e and d' express the influence of the residual orientation; f and f' are corrections to the origin of coordinates (z and z'). But since we express X and Y in minutes of arc and the measured coordinates x_m, y_m in millimeters, the coefficients d and e' for a normal astrograph (where 1mm $\sim$ 1) will be close to 1. Therefore we may set d = 1 + x, e = y, f = z, d' = x', e' = 1 + y', f' = z', where we shall take the small quantities x, y, z and x', y', z' as the unknown "constants" of the plate. Then the equations of condition (6) will be written as follows:

$$x_m + x_m \cdot x + y_m \cdot y + z - \frac{X_m}{\sin 1'} = 0$$

and

$$y_m + x_m \cdot x' + y_m \cdot y' + z' - \frac{Y_m}{\sin 1'} = 0$$

or denoting:

$$x_m = a, \quad y_m = b, \quad 1 = c, \text{ and } x_m - \frac{X_m}{\sin 1''} = n, \quad y_m - \frac{Y_m}{\sin 1'} = n'$$

we reduce them to the usual form of equations:

a.x + b.y + c.z + n = o; a.x' + b.y' + c. z' + n' = o (number of equations = m)

Both of these systems of equations are solved parallelly as they differ only by independent terms n and n' which are the comparisons between the coordinates measured on the plate and the ideal coordinates. Having obtained the values of the "constants", further computations are made according to the following formulae:

are the measured coordinates of the object under consideration).

$v_m = x_m \cdot x + y_m \cdot y + z; \quad v'_m = x_m \cdot x' + y_m \cdot y' + z'$. ($x_m$ and y_m are the measured coordinates of the object under consideration).

$$X_m = (x_m + v_m) \cdot \sin 1'. \qquad Y_m = (y_m + v'_m) \cdot \sin 1''$$

$$(\delta_m - D + \epsilon_m)' = y_m + v'_m - \frac{(y_m + v'_m)^3}{3} \cdot \sin^2 1';$$

$$(x_m + v_m) \cdot \sec(\delta_m + \epsilon_m) \cdot \cos(\delta_m - D + \epsilon_m) = \sigma_m$$

$$(\alpha_m - A)^s = 4 \cdot \left(\sigma_m - \frac{\sigma_m^3}{3} \cdot \sin^2 1'\right); \qquad \epsilon''_m = \frac{1}{2} K \cdot \sin 2\delta_m.$$

With the last formula it is possible to compute by logarithms the term of the third order correcting lg σ m according to the formulae

$$\Delta\, lg\, \sigma_m = mod. \frac{\Delta \sigma_m}{\sigma_m} = -mod. \frac{\sigma_m^2 . sin^2 1'}{3} = -\left[8.088_{-10}\right] \sigma_m^2 .$$

The universal and accurate methods of measurement and reduction of astrograms expounded above sometimes require a great expenditure of time. But there are cases when it is necessary to determine as quickly as possible the position of a single celestial object (comet, planet, variable star, etc) even if the accuracy has to suffer somewhat. In such cases it is often sufficient to interpolate the sought position of the object between two (or three) basic stars. The measurements are also simplified, they do not require a previous orientation of the plate. Such interpolation methods are those of Kaiser, Reger, Wolf, Schlesinger.

Let K be an object on the plate the position of which needs to be determined (Fig. 14). Let us select two basic stars: (1)and(2) such that object K be, as close as possible to the line (1) - (2). Let us pass through star (2) the celestial meridian and through star (1) the parallel of declination (we shall neglect the error of projection of the sphere on a plane). We measure the difference of the coordinates with respect to some axes X, Y: Δx and Δy for the pair of basic stars, Δx, Δy, between the object K and star (1), and Δx_2 Δy_2 between K and star (2). Let us denote the angles of inclination of lines (1) - (2) and the axis X to the parallel of declination by φ and θ respectively, and let us note that section (1) C = $\Delta\alpha$ and (2) C = $\Delta\delta$, where $\Delta\alpha$ is that difference of the right ascensions of the basic stars which was already reduced to the parallel, i.e. $\Delta\alpha = \Delta\alpha_0 . \cos 1/2\, (\delta_1 + \delta_2)$, where $\Delta\alpha_0 = \alpha_2 - \alpha_1$ on the equator.

From figure 14 we easily obtain the following relations:

$$tg\, \varphi = \frac{\Delta \delta}{\Delta \alpha};$$

$$tg(\varphi - \theta) = \frac{\Delta y}{\Delta x};$$

$$\Delta\alpha = (\Delta x . \cos\theta - \Delta y . \sin\theta) . s;$$

$$\Delta\delta = (\Delta y . \cos\theta + \Delta x . \sin\theta) . s. \qquad (11)$$

Factor S serves for the transfer from linear units, in which Δx and Δy are measured, to the angular ones, i.e. it expresses the scale of the photograph. Comparing object K with star (1) we similarly obtain:

$$\Delta\alpha_1 = (\Delta x_1 . \cos\theta - \Delta y_1 . \sin\theta) . s,$$

$$\Delta\delta_1 = (\Delta y_1 . \cos\theta + \Delta x_1 . \sin\theta) . s. \qquad (12)$$

$$\begin{aligned} \Delta\alpha_1 &= (\Delta x_1 . \cos\theta - \Delta y_1 . \sin\theta) . s ; \\ \Delta\delta &= (\Delta y_1 . \cos\theta - \Delta x_1 . \sin\theta) . s . \end{aligned} \qquad (12)$$

Eliminating factor S form equations (11) and (12) we have:

$$\Delta\alpha_1 = \frac{\Delta x_1 - \Delta y_1 . tg\theta}{\Delta x - \Delta y . tg\theta} . \Delta\alpha ; \qquad \Delta\delta_1 = \frac{\Delta y_1 + \Delta x_1 . tg\theta}{\Delta y + \Delta x . tg\theta} . \Delta\delta . \qquad (13)$$

But from the expression above we have:

$$tg\,\theta = tg\left[\varphi - (\varphi - \theta)\right] = \frac{tg\varphi - tg(\varphi-\theta)}{1 + tg\varphi . tg(\varphi-\theta)} =$$

$$= \frac{\frac{\Delta\delta}{\Delta\alpha} - \frac{\Delta y}{\Delta x}}{1 + \frac{\Delta\delta}{\Delta\alpha} . \frac{\Delta y}{\Delta x}} = \frac{\Delta x . \Delta\delta - \Delta y . \Delta\alpha}{\Delta x . \Delta\alpha + \Delta y . \Delta\delta} .$$

Substituting tgθ in the equations (13) we get after some calculation the following

$$\Delta\alpha_1 = \frac{\Delta x . \Delta\alpha + \Delta y . \Delta\delta}{(\Delta x)^2 + (\Delta y)^2} . \Delta x_1 - \frac{\Delta x . \Delta\delta - \Delta y . \Delta\alpha}{(\Delta x)^2 + (\Delta y)^2} . \Delta y_1 ;$$

and

$$\Delta\delta_1 = \frac{\Delta x . \Delta\delta - \Delta y . \Delta\alpha}{(\Delta x)^2 + (\Delta y)^2} . \Delta x_1 + \frac{\Delta x . \Delta\alpha + \Delta y . \Delta\delta}{(\Delta x)^2 + (\Delta y)^2} . \Delta y_1 .$$

Denoting the functions

$$\frac{\Delta x . \Delta\alpha + \Delta y . \Delta\delta}{(\Delta x)^2 + (\Delta y)^2} = a \quad \text{and} \quad \frac{\Delta x . \Delta\delta - \Delta y . \Delta\alpha}{(\Delta x)^2 + (\Delta y)^2} = b ,$$

We finally obtain

$$\Delta\alpha_1 = a . \Delta x_1 - b . \Delta y_1 ; \qquad \Delta\delta_1 = b . \Delta x_1 + a . \Delta y_1 ,$$

and likewise we obtain

$$\Delta\alpha_2 = a . \Delta x_2 - b . \Delta y_2 ; \qquad \Delta\delta_2 = b . \Delta x_2 + a . \Delta y_2 ;$$

The position of object K is obtained by the formulae:

$$\alpha_K = \alpha_1 + \frac{\Delta\alpha_1}{15}.\sec\frac{\delta_1+\delta_2}{2}; \qquad \delta_K = \delta_1 + \Delta\delta_1$$

or

$$\alpha_K = \alpha_2 - \frac{\Delta\alpha_2}{15}.\sec\frac{\delta_1+\delta_2}{2}; \qquad \delta_K = \delta_2 - \Delta\delta_2 .$$

It is evident that

$$\Delta\alpha_1 + \Delta\alpha_2 = \Delta\alpha, \text{ and } \quad \Delta\delta_1 + \Delta\delta_2 = \Delta\delta,$$

which gives us a check on the computations. This method of Kaiser ("flat" method), in case of large differences of the corrdinates between K and the reference stars, requires corrections for "the curving of the parallel of declination" and for "the convergence of the meridians", but we shall not go into that here.

F. Schlesinger's method requires three reference stars, so situated that the object to be measured is situated inside the triangle formed by the reference stars. In figure 15 let F_1, G_1, H_1 be reference stars, and A be the object to be determined. We orient the plate approximately with regard to the celestial axes and we measure accurately the rectangular coordinates of all the four objects, i.e. x_1, y_1, x_2, y_2, x_3, y_3, x_A, y_A; next, we draw our figure on paper and we continue line F A until it intersects line GH at point P; we measure the sections AP, FP, PH and GH. Then we take the ratios: $D_1 = \frac{AP}{FP}$ $D_2 = \frac{PH}{GH}$. $(1 - D_1)$ and $D_3 = 1 - D_1 - D_2$ and we form the following expressions:

$$\left.\begin{aligned} m_x &= x_A - (D_1.x_1 + D_2.x_2 + D_3.x_3); \\ m_y &= y_A - (D_1.y_1 + D_2.y_2 + D_3.y_3). \end{aligned}\right\} \qquad (14)$$

The quantities D_1, D_2, D_3 are called "dependencies" and Schlesinger offers the following theorem: "If the figure and the measurements were quite exact, we would have: $m_x = 0$ and $m_y = 0$". This statement corresponds to the situation in which A' is the center of gravity of point F, G, H when they are given correspondingly equal weights: D_1, D_2, D_3 $(D_1 + D_2 + D_3 = 1)$. This theorem may be easily proven for m_x if we project the sections indicated above on the axes of th the coordinates, and, noticing that:

$$\frac{AP}{FP} = \frac{\xi - x_A}{\xi - x_1} \qquad \text{and} \qquad \frac{PH}{GH} = \frac{x_3 - \xi}{x_3 - x_2},$$

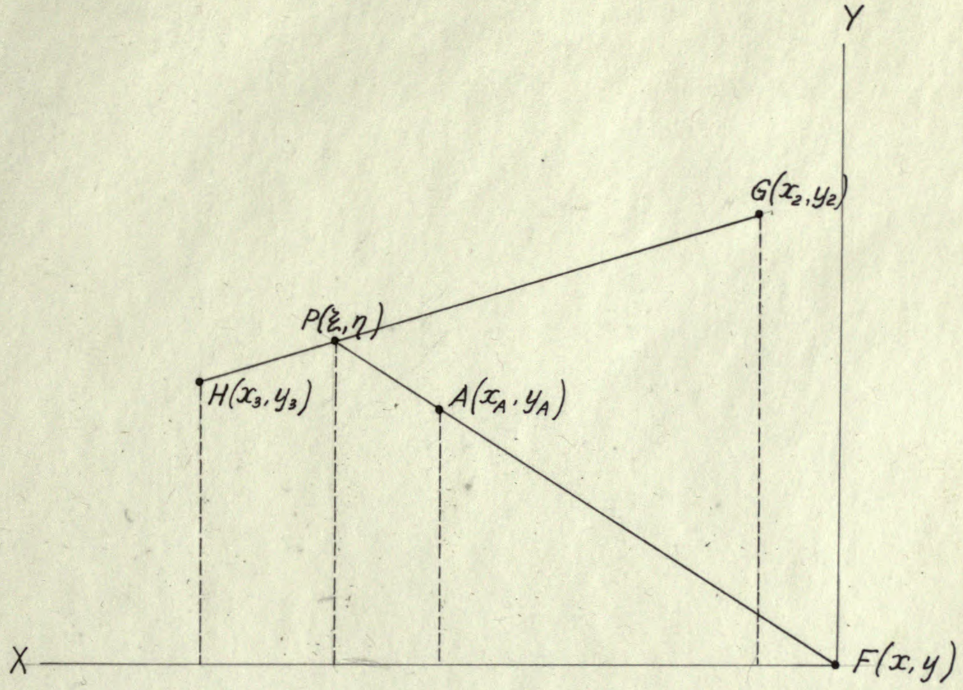

Fig. 15.

where ξ, η are the coordinates of point P_1 let us enter these values in the formulae for the "dependencies", and then into equations (14). The proof for m_y is quite analogous.

The next step in Schlesinger's idea is as follows: many volumes of the "photographic catalogue of the sky" are already published where accurate rectangular coordinates are given for all stars up to the eleventh magnitude (see next section § 4); also tables (or "constants") are published for the transformation of rectangular coordinates into equatorial ones. If our plate is taken by a normal astrograph i.e., on the same scale as those of the photographic catalogue, it is not difficult to find a plate among those of the catalogue close enough to ours in regard to the position of the optical center. Identifying there (by their coordinates) our basic stars F, G, H and having written their exact coordinates X_1, Y_1, X_2, Y_2 and X_3, Y_3, for 1900.0 we may write:

$$\left.\begin{aligned} X_A &= D_1 . X_1 + D_2 . X_2 + D_3 . X_3 + m_x ; \\ Y_A &= D_1 . Y_1 + D_2 . Y_2 + D_3 . Y_3 + m_y , \end{aligned}\right\} \qquad (15)$$

as the previous reasoning and proof do not depend on the position of the axes of the coordinates. Here X_A, Y_A are the coordinates of the object under determination in the system of the photographic catalogue (for 1900.0).

Transfering the origin of the coordinates to the point ($X_A = Y_A = 0$) and connecting equations (14) and (15) we obtain

$$X_A = D_1 . (X_1 - x_1) + D_2 . (X_2 - x_2) + D_3 . (X_3 - x_3) ;$$
$$Y_A = D_1 . (Y_1 - y_1) + D_2 . (Y_2 - y_2) + D_3 . (Y_3 - y_3) .$$

Having computed X_A and Y_A according to these formulae, we then transform them into the ideal coordinates with the aid of constants which are given in catalogues, and finally into α and δ by known formulae.

The described method has the great advantage of enabling us to select reference stars up to the eleventh magnitude, i.e. a much bigger selection, than in the visual catalogues. It is a transfer of the photographed plate to the plate of the photographic catalogue and it would be entirely accurate if the optical centers of the two plates were to coincide. Unfortunately, this does not generally happen, and the plates are inclined to one another (up to 1°) and therefore a correction must be introduced which reaches sometimes 0".5.

The graphic method of the determination of "dependencies" is less accurate, but a computational method may be offered for this purpose. Indeed, Schlesinger's method assume a linear dependence of the measured coordinates x_1, x_2, x_3, y_1, y_2, y_3 on the plate on those on the plate of the photographic catalogue, therefore we may write:

$$\begin{aligned} d x_1 + e . y_1 + f &= X_1 \\ d . x_2 + e . y_2 + f &= X_2 \\ d . x_3 + e . y_3 + f &= X_3 \end{aligned}$$

and

$$X_A = d . x_A + e . y_A + f .$$

Solving this equation by means of determinants, we obtain

$$f = \frac{\begin{vmatrix} x_1 & y_1 & X_1 \\ x_2 & y_2 & X_2 \\ x_3 & y_3 & X_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} = \frac{\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} . X_1 - \begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} . X_2 + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} . X_3}{\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} - \begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}}$$

or denoting the minors as follows:

$$+\begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} = M_1 , \quad -\begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} = M_2 \text{ and } +\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = M_3$$

we have

$$f = \frac{M_1 . X_1 + M_2 . X_2 + M_3 . X_3}{M_1 + M_2 + M_3}$$

But transfering, as we had previously done, the origin of the coordinates into point A, we shall obtain $X_A = f$, i.e.

$$X_A = \frac{M_1 . X_1 + M_2 . X_2 + M_3 . X_3}{M_1 + M_2 + M_3}$$

and comparing this equation with the first of the formulae (15) we obtain

$$D_1 = \frac{M_1}{M_1 + M_2 + M_3} ; \; D_2 = \frac{M_2}{M_1 + M_2 + M_3} ; \quad D_3 = \frac{M_3}{M_1 + M_2 + M_3} .$$

2. The Principal Application of Photographic Astrometry.

In the present subchapter we shall speak only of the main applications of photographic astrometry in stellar astronomy, namely: the mapping of the sky and the composition of stellar catalogues by means of photography, and the determination of annual parallaxes of stars and their proper motions. Because of lack of space we speak mostly on methods of determination, touching but slightly the results obtained referring those who are interested in the details to the special literature.

§ 4. Photographic Maps and Catalogues. a) The Photographic Catalogue of the Southern Sky. The well known stellar catalogue and atlas of the sky - the Bonner Durchmusterung, - composed visually in the fifties of the last century by Argelander and continued by Schonfeld, gives approximate positions (to $0^s.1$ and $0'.1$) of all stars up to 9.5 magnitude, from the north pole down to 23° of southern declination, altogether about 458000 (for 1855.0)*. The continuation of this Durchmusterung to the sough pole of the sky was made from 1885 to 1891 at the Observatory of the Cape of Good Hope, photographically according to the idea of David Gill. An astrograph was used, consisting of a camera of short focus with a Dallmeyer objective (152mm in aperture) and of a guiding tube with an objective of 76mm having a focal length of 1.4m. Altogether 613 plates of an area of 36 square degrees each with an exposure lasting 60 minutes were made.

It was originally ntended to publish the results of this work in the form of a photographic atlas of the southern sky, but then the entire material was turned over to J. C Kapteyn at Groningen under whose supervision a great amount of work was accopmlished by means of a measuring apparatus specially constructed (by Kapteyn) for this purpose. The work continued for 5-6 years and it consisted in measuring and reducing all the plates and in cataloguing all the stars almost up to the tenth magnitude. As a result a photographic catalogue - the Cap Photographic Durchmusterung - in three volumes was published in 1900 containing positions (for 1875.0) and photographic magnitudes of more than 400000 stars.

b) International Photographic Map and Catalogues of the Entire Sky. The great progress in the development of photography and its applications reaches in the seventies and eighties of the last century (see § 24) led to the idea of a systemetic photographing of the sky. The excellent astrophotographs obtained by the Henry brothers. (Paul and Prosper Henry), Parision astronomers, greatly enhanced the idea. In 1887 the idea became so widespread that in April of the same year admiral Mouchez, the directer of Pairs Observatory, called the first astrophotographic conference in which representatives of 16 European and non European countries participated (among them also representatives from Pulkovo). Similar conferences were held in 1889 and 1891 and in following years; a permanent committee was formed to faciliate the realization of the great undertaking to create a photographic map and a catalogue of the entire stellar sky (Comite inter-

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* The continuation of the Durchmusterung for the southern sky was made at the observatory of Cordoba (Argentine) under the direction of J. Thome; after his death in 1908 the work was continued by his widow and at the present time the work is nearing completion. As a result the so called Cordoba Duchmusterung (atlas and catalogue) will contain stars from 23° to 90° in declination - altoghther a about 650000 stars up to 10th magnitude.

national permanent pour l' e ex'cution de la Carte photographique du Ciel), and the following plan of work was worked out.

1) All photographs must be of a single type (in the sense of optical constant of the astrographs) and the instrument of the Henry Bros.* which was subsequently called the "normal" astrograph was taken as the standard (see Fig. 16 and the description of the normal astrograph of the Pulkovo Observatory in §1 of this chapter);

2) two series of photographs should be taken which will cover the entire sky: the first series with an exposure of about 60 minutes in order that stars u up to the fourteenth magnitude may be obtained, and the second series with an exposure of 5 - 6 minutes so that all stars up to the eleventh magnitude may be obtained. The photographs of the first series magnified twich and printed on paper were to constitute the "photographic map" of the sky containing more than 20 million stars. The photographs of the second series were to be measured by accurate instruments and to constitute a catalogue of accurate positions of two million stars up to the eleventh magnitude approximately (for 1900.0) with an indication of their photographic magnitude;

3) in each of the two series the entire sky is to be covered twice, and the photographs are to be so distributed that the optical centers of the plates surrounding each of them shall be close to the corners of the latter; thus each star is photographed twice and on the average in favorable position.

As a plate of a normal astrograph covers about four square degrees on the sky, the entire number of pl tes for each series is 22054. The entire work was originally distributed among 18 observatories (12 in the northern and 6 in the southern hemisphere) but changes were made later as some observatories had to give up their portion of the work because of a lack of funds. This fact retarded the publication of the "photographic maps" (up to 1915 only 3600 of them were published), and in later years more attention has been given to the measurement of plates of the second series for the "photographic catalogue" and to the publication of its separate zones. Only the "rectangular coordinates" of all stars up to the eleventh magnitude are mostly published, and only a few observatories have completed the work up to the computation of the final equatorial coordinates (α and δ for 1900.0).

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*) The Henry Bros. were well known opticians who polished their own lenses and who built a large astrograph which became the prototype of of all instruments of this kind.

Table 1 shows the distribution of various zones among the various observatories*); also indications are given as to which part of the photographic catalogue were published up to 1930 and the form in which they were published. This is of practical importance for various astrophotographic work; for instance, in the case of selection of comparison stars in the application of Schlesinger's method, etc.

Table 1, see p. 23

As may be seen from Table 1, only a few observatories give final, accurate equatorial coordinates (α and δ for 1900.0) fo the stars, most of the observatories published only their corrected rectangular coordinates; some for the transformation of the measured rectangular coordinates into "ideal" coordinates and sometimes tables are also given for the transformation of the latter into α and δ. It is necessary to note that the α and δ given in some catalogues and the "constants" of plates given in other catalogues are based on the positions of reference stars taken by various observatories from different sources (A. G. Zones, old and new catalogues, special series of meridian observations, etc); besides the methods of reduction used were different in various observatories. The result is that the α and δ given in various zones are related to different systems, and, strictly speaking, it will be necessary sometime in the future to undertake the colossal task of recomputing all constants and to produce a general photographic catalogue of the entire sky - which will be based on a general and uniform catalogue of reference stars obtained by a definite system.

To be sure, this problem was posed from the vary beginning of this entire great work, but only at the fifth international astrographic conference were real efforts made in this direction. The Pulkovo Observatory was represented at this conference by its director O. A. Baklund. On the report of J. Kapteyn the conference decided that "in the future all the regular meridian observations, except those for special investigations, are to be limited to observations, of those stars which are reference for the photographic catalogue". It was decided to use the usual chain method of determinating stellar position, namely all the stars to be determined are divided into the three groups which are consecutively connected to each other:

1) Reference stars so selected that one star corresponds to each 25 square degrees of the sky (their number is 1610). Some of the most important observatories (Pulkovo, Greenwich, Paris, Cape of Good Hope, Washington, etc) have undertaken the reobservation and development of this catalogue in which were

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*) The Pulkovo normal astrograph was built in 1893 and systematic work with it started only in Sept. 1895. Therefore, the Pulkovo Observatory did not participate directly in the work under our consideration, but it helped this work indirectly to a great extent.

Table 1

Observatory	Geographic Latitude	Declination Zone		What was published
		From	up to	
Greenwich	+ 51°	+ 90°	+ 65°	All is published in full: rectangular coordinates and α and δ.
Rome (Vatican)	+ 42°	+ 64°	+ 55°	Rectangular coordinates for the entire zone.
Catania	+ 37°	+ 54°	+ 47°	Rectangular coordinates for the entire zone; α and δ only in part.
Helsingfors	+ 60°	+ 46°	+ 40°	Rectangular coordinates and accurate α and δ for all hours of right ascension except 18^h - 21^h and $22^h\ 30^m$ - 24^h.
Potsdam	+ 52°	+ 39°	+ 32°	Rectangular coordinates for the entire zone; α and δ only approximately.
Oxford	+ 52°	+ 31°	+ 25°	Rectangular coordinates for the entire zone.
Paris	+ 49°	+ 24°	+ 18°	Rectangular Coordinates for the entire zone, except + 18° of declination.
Bordeaux	+ 45°	+ 17°	+ 11°	Rectangular coordinates for the zone + 12 - + 17°
Toulouse	+ 44°	+ 10°	+ 5°	Rectangular coordinates for zones + 5°, + 7°, + 9°, + 10° - + 12°.
Algiers	+ 37°	+ 4°	- 2°	Rectangular coordinates in full for the entire zone.
San Fernando (Spain)	+ 36°	- 3°	- 9°	Rectangular coordinates for the zone - 3° to - 8° in declination.

Table 1 Cont'd

Observatory	Geographic Latitude	Declination From	up to	What was published
Tacubaya (Mexico)	+ 19°	- 10°	- 16°	Rectangular coordinates and α and δ for the zone from 14° to - 17° in declination.
Hyderabad (India)	+ 17°	- 17°	- 23°	Rectangular coordinates for the zone from - 17° to-22°.
Cordoba (Argentine)	- 31°	- 24°	- 31°	Rectangular coordinates for the zone from - 24° to - 26°.
Perth (W. Australia)	- 32°	- 32°	- 40°	Rectangular coordinates for - 32° in declination.
Cape of Good Hope	- 34°	- 41°	- 51°	Rectangular coordinates and α and δ in full for the entire zone.
Sidney (Australia)	- 34°	- 52°	- 64°	Rectangular coordinates for the zone - 53° and - 52° (in part).
Melbourne (Australia)	- 38°	- 65°	- 90°	Rectangular coordinates for the zone from - 65° to - 68°.

entered 1200 of the so called Pulkovo "fundamental" stars;

2) intermediate stars, the number of which is 10.000 over the entire sky; and, finally;

3) proper referenct stars the number of which is larger than 100.000, i.e., on the average from 10 to 20 stars from each plate of a normal astrograph.

Unfortunately, the imperialist was which began in 1914 hindered the systematic execution of this plane and, in general, slowed up the entire work of the photographic catalogue and the maps of the sky; only the previously mentioned catalogue of reference stars may be considered as wholly finished (in Pulkovo and catalogue is for 1915.0). In conclusion we shall note in regard to the accuracy of stellar positions in photographic catalogues that the best of them, the Helsingfor's catalogue, contains a possible error of only +00^{s}.01 in α and +0."1 in δ for the final mean position of each star; in other catalogues the error is somewhat larger - from +0".1 to +0".2 in declination.

c) The Photographing of the A G Zones in connection with the necessity of accurate determinations of positions of reference stars let us mention the astrographic work in this direction which is also being carried on at the present time. The "Astronomische Gesellschaft" established in Germany in 1863 selected for its first problem the determination by transit instruments of accurate positions of all stars up to the 9th magnitude inclusive which are registered in the Bonner Durchusterung. As a result of a very great effort distributed among many observatories in several countries and which lasted for many years, the well known catalgue of A G Zones come into existence containing about 138,000 stars within -2° to +80° in declination and giving α and δ for 1875.0 within 0^{s}.01 and 0".1. (The A G catalogue is also being continued gradually southward). This catalogue has already become the basis for many zones of the International Photographic Catalogue and for other astrographic work, but there is no doubt that at present it is not sufficiently accurate (the mean possible error of position of a star is +0".50 in declination and a corresponding value in right ascension) as well as because its epoch of observation is already far away. We do not know the proper motions of major part of the stars contained in this catalogue, and for this reason the error in the position of a star in the last fifty years may reach several seconds of arc.

All this led to the idea that the zones of A G need be observed again, at least, for the northern sky, and it was natural to think of the new powerful photographic method in order to save time andeffort and to combine the photographic method with simultaneous observations of fundamental stars with transit instruments. It was evident that because of the comparative brightness of stars to be determined (9th or 10th magnitude) and of the "mass-production" work, it w was best to use comparatively small photographing insturments of smaller light gathering power, yet of such optical quality that it might be possible to use

larger plates covering a larger portion of the sky in angular measure.

1). The first experiments in this direction were made by Schlesinger at the Alleghany Observatory in 1914, and the good results obtained by him led the Astronomische Gesellachaft to consider the problem seriously. After consideration of this problem at several conventions (in 1922, 1924, 1926) a detailed program was worked out the main points of which are as follows:

1) The new fundamental catalogue of Auwars (F K^3) is taken as base and to it are added a special catalogue of 13.755 fundamental stars of new observations by transit instruments composed by Kustner. Beginning in the winter of 1927-28 these observations will be made in full by the Berlin - Babelsberg and the Hamburg - Bergedorff observatories; besides, the observatories in Breslau, Pulkovo, Bonn, Leipzig and Heidlberg will take on some work in special zones.

2) A special type of zone astrograph was developed (made by Zeiss in Jena) of the following dimensions: a wide-angled objective of 160mm apperture (central diaphrams is 85mm) with a principal focal length of 206cm. (Scale: 100" in 1mm); the plates are in plate glass specially investigated for a "plane" of 25x25 cm with an utilized field on the sky of about 5°x5°.

3) The taking of the photographs was entrusted to three observatories by a division of the northern sky into three zones, namely: Pulkovo (from +90 to 70° in declination), Hamburg-Bergedorf (from +67°.5 to +22°.5) and Bonn (from +20° to 2°.5. The neighboring plates should overlap by half (as was done in the international map of the sky) and on each plate is made two photographs: one of the exposure of 10-15 minutes and the other of 3 minutes; the emulsion is of the highest sensitivity (Astro-Platten of the Agfa firm).

4) The measurements of the plates in rectangular coordinates are made on specially constructed instruments (Bamberg's make) of long micrometric screw and with the application of a reversing prism. Besides the stars in the zone catalogue additional stars are measured (from the Kustner and the Carrington catalogues).

5) The measured coordinates corrected for instrumental errors are reduced to 1950.0 taking into consideration terms of the second order of the refraction. The final computation of equatorial coordinates and the compilation of the catalogue will be done by the "Astronomisches Rechen - Institut, Berlin-Dahlem.

As a result of this well orgainzed and rapidly progressing work we shall soon have good positions of about 150.000 stars (with a possible error of about +0".2 in declination) and their proper motion derived from a comparison with the AG Catalogue for 1875.0. The problem of a similar work to cover the southern sky remains as yet an open one.

d) Special Work on the Cartography of the Sky. It is necessary to mention two more photographic works on the cartography of the sky. They are of importance for practical work and for stellar statistics especially singe the international

map of the sky is appearing very slowly. We have in mind the Photographic Atlas of the entire sky which is being published by the Greenwich Observatory and based on 206 photographs made in 1903-1910 by an amateur in astronomy I. Franklin - Adams. These photographs were obtained by him in England and in the Transvaal by a Cook triplet (d = 25cm, F = 1.1m) on plates 45x45cm with an exposure of 2 hours and 20 minutes and they contain stars up to the 15th magnitude. A second work of the same nature but limited to the zodiacal zone of the sky (in connection with the search for minor planets) was done by M. Wolf and I. Palisa through photographs made by the well known "Bruce" - telescope of the Heidelberg Observatory (astrograph with two cameras: d = 40cm, F = 2m); 200 maps have been published so far with a field of 6°x7° on a scale of 2' per 1mm containing stars down to 16th magnitude, approximately.

§ 5. Determination of Stellar Parallaxes by the Photographic Method.

The annual parallax of stars is one of the most important stellar characteristics which gives us a notion of the distances of stars from our solar system and of the true dimansions of the universe. At the same time the determination of the quantity trigonometrically is a very difficult problem in astromony as may be seen from the following considerations:

1) we know from direct measurements and from statistical considerations that only a very few stars are comparatively close to the solar system and that the annual parallaxes of stars are, on the average, very small and are almost on the border of the limits of accuracy of our measurements (the parallax of the nearest star, α Centauri is 0".76, that for second nearest star, Proxima Ophiuchi (17^h53^m + 4°25') is 0".54, and for stars of the eighth magnitude the parallax is, on the average of the order of 0".008;

2) the influence of the parallax of a star on its apparent coordinates on the sky is a function with a period of one year (this is why we say "annual parallax"), but most of the systematic errors of the measurements depend on the harmful influence of temperature changes upon our instruments, the accurate measuring instruments, atmospheric conditions etc.; hence these errors have also an annual period and in many cases they may conceal entirely the influence of the parallax.

This is the reason why astronomers long ago gave up the methods of determination of absolute parallaxes of stars and apply instead the differential methods only, i.e. comparing the annual parallaxes of neighboring stars the positions of which are subject to more or less the some harmful influences (instrumental, atmospheric, etc) and hence they concel each other. Thus a very accurate relative parallax of a star is obtained, i.e. the difference between

the absolute parallaxes of a given star and thd comparison star. If there are grounds for assuming that the comparison star is very distant (faintness, absence of noticeable proper motion, etc), the absolute parallax may be taken as practically zero, or it may be deducted from other, statistical considerations. Thus, we may come close to the absolute value of the parallax of the star under consideration.

Numerous methods of observation have been applied for the determination of trigonometric parallaxes, such as: micrometric measurements with refractors, heliometric measurements and, finally, differential observations of transits of stars across the meridian. But all these methods are now supplanted by the much more accurate photographic method. Let us note that other indirect methods are also used at present (of which a detailed account will be given in another chapter), for instance: statistical, photometric, dynamic and, especially, spectroscopic methods, but we must say that the basis of many of these methods are parallaxes of individual stars which were obtained trigonometrically; thus, systematic work in this field is necessary.

We shall expound here Kapteyn's mass determinations (Durchmusterung) of stellar parallaxes upon a single plate. Though this method is rarely used nowaday, many investigations and remarks are connected with it which are common to all other contemporary methods. We shall next expound the more modern photographic methods used for the determinations of parallaxes of separate stars.

a) Kapteyn's Method. As far back as 1892 Kapteny expressed the original idea about the possibility of using the "hidden image" in order to mark the photographic method of determinations of parallaxes (or of proper motions) differential. The idea is as follows: Let us make a photograph of the region of the sky under investigation at an epoch when this region procedes the Sun by about 6^h in right ascension (Phot. 1 M on Fig. 17)*. As it is geometrically easy to imagine, the annual parallax of stars at this epoch and in the given region will have a maximum influence on the stellar right ascension by increasing it. Obtaining thus a hidden image 1 M for a series of stars S_1, S_2, S_3 . . . we leave the plate undeveloped for about 6 months i.e. up to the epoch when the visible right ascension of the Sun changes by 12^h, and the region of the sky under investigation will follow after it approximately by 6^h in right ascension. Placing then our plate again upon the instrument in the same way and merely shifting the image by several seconds in α, we take another photograph (m) of the same region of the sky under the same conditions as much as possible. Then shifting the images on the plate a little more we obtain a third photograph (m) and we again leave the plate undeveloped for six months. Finally, we make the last fourth photograph (M) - almost a year after the first photograph; then the photograph is developed in the usual way.

Assuming that the photographs are taken near the meridian, it is evident that, in order to weaken the influence of the "hour angle errors", the photograph 1 and 4 will be made in the morning before sunrise, the photographs 2 and 3 are made in the evening after Sunset (the latter two photographs are only a few minutes apart).

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* See Sheet 38-a

Other arrangements of the photographs are possible, but the scheme on Fig. 17 is the most typical.

In the ideal case when the phtotgraphs are situated not in right ascension but in longitude and are made at the ecpchs of maximum positive or negative influence of the parallax, it is evident geometrically that the distance P between the first and second image will be shortened for each star by 2π where π is its absolute parallax, and distance Q between the 3rd and 4th photographs are lengthened by 2π . Thus the difference $Q - P$ will contain, besides all other influences, the quadrupled amount of the annual parallax, and the sum $Q + P$ will be free of the influence of the parallax, but will contain the influence of the proper motion for one year. It is more convenient in practice to do as was said above, so that the coefficients of the influence of the parallax are always a little less than unity.

Let us pass to the problem of measurement and reduction of the plate photographed by Kapteyn's method. Orienting the plate in the instrument, the distances P and Q for all stars appearing on the negative are accurately measured by the micrometer scres as well as (approximately) the rectangular coordinates x and y of stars with respect to the optical center of the plate. The measurements are made as usual in two positions of the plate differing by 180° in order to exclude the personal error of setting. Let us note that the technical errors of the instrument have no influence hare because of the smallness of the measured distances (of the order of several tenths of a millimeter); besides Q and P change little from star to star, so that in the comparison of the stars with each other all technical errors are excluded from the result.

Let us denote now by x_1y_1, x_2y_2, x_3y_3, and x_4y_4 the rectangular coordinates of images 1 ... 4 taken in the epoch t_1, t_2, t_3, and t_4. Then let π be the absolute parallax of the given star, and $\mu_x = \mu_\alpha \cdot \cos\delta$ be its proper motion along the coordinate X; let us denote, finally, by h the coefficient of influence of the parallax along coordinate X determined by the well known formulae of spherical astronomy:

$h = m \cdot R \cdot \sin(\odot - M)$; $m \cdot \sin M = \sin \alpha$; $m \cdot \cos M = \cos\alpha \cdot \cos$; $h \cdot \sec\delta = \alpha_{vis} - \alpha_{true}$,

where $\odot$ is the longitude of the Sun, R is the radiusvector of the earth's orbit, is the inclination of the ecliptic and α and δ are the optical center of the plate. Passing from the apparent measured coordinates to the true ones with respect to some fixed system of axes, we must take into consideration the influence of the following factors:

1) the correction of the accepted value of the scale of the plate;

2) the orientation and transfer to a new origin of coordinates (the influence of precession and nutation changing only the position of axes are included here);

3) normal and abnormal distortion of the objective;

4) other instrumental errors which are functions of the position of the image of the star on the plate (for instance, the inclination of the plate to the optical axis, etc.).

5) differential refraction and aberration;

6) the annual parallax and proper motion of a star.

A substantial part of all these factors except (6) may be expressed by a linear function of the coordinates x and y, and only for some terms of the second order (refraction for instance) may be needed which are usually small when the proper precautions are taken. Therefore we may write for our four images of a given star the following:

$$1)\quad \xi_1 = x_1 + a_1 + b_1 . x_1 + c_1 . y_1 + d_1 . x_1^2 + e_1 . x_1 . y_1 + + f_1 . y_1^2 - h_1 . \pi - t_1 . \mu_x$$

$$2)\quad \xi_2 = x_2 + a_2 + b_2 . x_2 + c_2 . y_2 + d_2 . x_2^2 + e_2 . x_2 . y_2 + + f_2 . y_2^2 - h_2 . \pi - t_2 . \mu_x$$

$$3)\quad \xi_3 = x_3 + a_3 + b_3 . x_3 + c_3 . y_3 + d_3 . x_3^2 + e_3 . x_3 . y_3 + + f_3 . y_3^2 - h_3 . \pi - t_3 . \mu_x$$

$$4)\quad \xi_4 = x_4 + a_4 + b_4 . x_4 + c_4 . y_4 + d_4 . x_4^2 + e_4 . x_4 . y_4 + + f_4 . y_4^2 - h_4 . \pi - t_4 . \mu_x .$$

As the coordinates x_1y_1 .... x_4y_4 differ little one from the other and the coefficients b_1 c_1 d ..... are small quantities, we may take for each star the coordinates of the center of gravity of all its images, i.e. we may assume

$$\frac{x_1 + x_2 . x_3 + x_4}{4} = x$$

and

$$\frac{y_1 + y_2 . y_3 + y_4}{4} = y .$$

Taking this into consideration we subtract the first equation from the second, and the third from the fourth, and we connect all terms of the first order into one linear function while the terms of the second order are set separately. Then we have:

$$\left.\begin{aligned} \xi_2 - \xi_1 = {} & x_2 - x_1 + \alpha_1 + \beta_1 . x + \gamma_1 y_1 + \\ & + \varphi(x,y) - (h_2 - h_1)\pi - (t_2 - t_1)\mu_x \\ \\ \xi_3 - \xi_4 = {} & x_4 - x_3 + \alpha_2 + \beta_2 x + \gamma_2 y_2 + \\ & + \psi_2(x,y) - (h_4 - h_3)\pi - (t_4 - t_3).\mu_x \end{aligned}\right\} \quad (16)$$

where φ_2 and ψ_2 denote the sum of all terms of the second order. Noticing that $x_2 - x_1 = P$ and $x_4 - x_3 = Q$ and that the differences of the true coordinates $\xi_2 - \xi_1$ and $\xi_4 - \xi_3$ are constant quantities for all stellar images (a genaral shift of images on the plate), we connect these latter with the constants α_1 and α_2 and then we exclude from equation (16) the proper motion μ_x. As a result we obtain

$$\begin{aligned} & P . \frac{t_4 - t_3}{t_2 - t_1} - Q + A' + B' . x + C' . y + \varphi_2 - \\ & - \left[(h_2 - h_1) . \frac{t_4 - t_3}{t_2 - t_1} - (h_4 - h_3) \right] \pi = 0 \end{aligned}$$

Denoting here

$$\frac{t_4 - t_3}{t_2 - t_1} = \theta, \quad \text{and} \quad (h_2 - h_1) . \theta - (h_4 - h_3) = H,$$

we finally obtain

$$A + B . x + C . y + \varphi_2 + H . \pi + Q - \theta . P = 0. \qquad (17)$$

On the other hand, adding the equations (16) we obtain

$$D + E.x + F.y + \psi_2 + (t_2 - t_1 + t_4 - t_3).\mu_x + \\ + (h_2 - h_1 + h_4 - h_3).\pi - (P + Q) = 0.$$

As $t_3 \sim t_2$ and $h_3 = h_2$, the second and third photographs fall out, and, when the first and fourth photographs are close to each other in the same epoch of the year, then $h_1 \sim h_4$, i.e. the coefficients at π turn into zero and the equation assumes the form

$$D + E.x + F.y + (t_4 - t_1).\mu_x - (P + Q) = 0. \qquad (18)$$

Here P + Q may directly represent the distance between the first and last photograph (along the coordinate x) the difference in the epochs ($t_4 - t_1$) of which may be increased by many years on special photographs. The influence of errors of the second order is almost excluded when both photographs are made close and at equal hour-angles ($\psi_2 \sim 0$).

Turning to formula (17) and neglecting for a while the terms of the second order φ_2 which may be determined separately or computed a priori (for refraction), we note that the quantities x_1 y_1 H and Q - $\theta.P$ x P are known from the measurements and computations, but the coefficients A_1 B_1 C_1 constants for all stars on the entire plate and the absolute parallaxes $\pi_1, \pi_2 \ldots\ldots \pi_m$ (if m stars are measured on the plate) are unknown. Therefore, there are m equations of conditinn of the form (17) with (m + 3) unknown s, and it is clear from this that we may determine only the relative values of the parallaxes by solving this system of equations by successive approximations.

If we have no previous data in order to judge the magnitude of the parallax of this or that star from the number of stars measured on the plate (brightness, large proper motion, etc). The most natural hypothesis would be the initial assumption that all absolute parallaxes $\pi_1, \pi_2 \ldots\ldots \pi_m$ are practically equal to zero, or, more correctly they are within the limits of errors of observation and measurement. Then we shall have a system of m equations with three unknowns: A, B, C of the form

$$A + B.x + C.y + Q - \theta.P = 0.$$

Solving this system by the method of least squares, we may compute the function

$$v = A + B.x + C.y$$

for all measured stars, and substituting its value into equation (17), we obtain therefrom for each star;

$$\pi = -\frac{v + K}{H}, \qquad (19)$$

where $K = Q - \theta . P$ (or $K = Q - \theta . P + \varphi_2$)

In other words, we compute the remaining errors v + K in conditional equations and we try to find there noticeable parallaxes. If we find them, the equations of condition for the respective stars are excluded and a second computation by the method of least squares is made (second approximation).

It is not difficult to see that each parallax computed by formula (19) is a relative one, taking the mean parallax of all stars entering into the system of conditional equations as zero. Indeed, summing up the fundamental equation (17) for all m stars we have

$$m . A + B . [x] + C . [y] + H . \Sigma \pi + [K] = 0 .$$

But in the determination of the coefficients A, B, C by the method of least squares the first normal equation has the form:

$$m . A + B . [x] + C . [y] + [K] = 0$$

and connecting it with the foregoing we have

$$H . \Sigma \pi = 0, \quad \text{or} \quad \Sigma \pi = 0, \text{ and } \frac{\Sigma \pi}{m} = 0$$

Hence it follows that, theoretically, the probable relative parallax should be with a plus (+) sign for about half of all stars measured on the plate, and with a minus (-) sign for the other half; but it is evident that all stars with a noticeable parallax will have a tendency to be in the negative region; this circumstance is important statistically.

As to the terms of the second order $\varphi_2(x, y)$ the investigations of several scientists (Kapteyn, Von-Mannen, anf others(have shown that if some precautions are taken (photographs near the meridian, weakening the brightness of the bright stars by various devices, etc) they are very small, and they may be detected as systematic errors only by comparing the results obtained by different methods.

Kapteyn's method has been successfully applied at several observatories (Helsingfors, Pulkovo, Bonn, etc) yielding individual parallaxes with a possible error from +0".01 to +0".03 (for a normal astrograph); using instruments of long focus and in better atmospheric conditions the possible error should become still less. However, at present, due to technical causes, it is preferable to make photographs at the epoch of maximum influence of the parallax on separate plates even for the determination of the parallax, and then they are coordinated by methods which we shall describe.

b) General Methods of the Determination of the Parallax of an Individual Star from a Series of Plates. All these methods are based on the comparison of a series of plates with one or several photographs of that part of the Sky where the star under consideration is located. Usually we obtain from 10 to 20 plates, taken in groups of 3-5 plates near the epochs of the maximum influence of the

parallax in right-ascension (or in longitude); hence all observations are distributed, approximately, through 1 1/2 - 2 years (sometimes longer). The plates are developed immediately after photographing and then they may be measured by various methods (in polar coordinates, rectangular coordinates, comparison of plates in pairs or with one of them, etc.). Before we go into detailed discussions in photographing comparatively bright stars in order to obtain their parallax. We have previously said that in a simultaneous photographing of faint and of bright stars on a plate harmful effects may appear having the character of a "light equation" and distorting the relative postions of their photographic images. Here belong the so called "hour angle errors" depending on the inaccuracy of the setting of the astrograph, on the influence of differential refraction and atmospheric dispresion, etc; then the guiding errors acting on the right ascension, reflecting differently on stars of various brightness and depending on the not quite correct hour hemchaism as well as on the inaccuracy of the "perception" on the guide star (observer's personal error). It is evident that for the exclusion of the "light equation" it is necessary to compare the brightness (magnitude) of the photographic image of the star the parallax of which is determined with the mean brightness of the comparison stars which are selected usually so as to be not brighter than a star of the 10th magnitude. This can be achieved by several methods;

1) The central part of the plate where the image of the star is located is treated by a chemical substance which lowers its sensitivity (for instance: by a 1°/o solution of copper sulphate).

2) A simitransparent screen absorbing a definite part of the light of a star under investigation is put on the central part of the plate right on top of the sensitive layer.

3) Immediately in front of the center of the plate an opaque screen is placed which has the form of a small disk with a sector cut out of it. The angle of this sector may be changed within wide limits: the entire disk rotates during the exposure with a velocity of 6 - 8 rotations a second (by means of a small electric motor). It is evident that the exposure of the star, the parallax of which is b being determined, by broken up into a series of short periods, which in their sum produce such an influence of the plate as if the brightness of the star decreased in the ratio of the angle of the sector to 360°; thus it becomes possible to lower the brightness of a star by 6 - 7 magnitudes.

4) In front of the objective of the tube a diffraction grating is placed consisting of a series of parallel wires. As a result the plate gets, besides the central image of each star, also on each side of the central image two (or more) diffraction images in the form of very small spectra which are considerably less bright then the central images. It is evident that in measuring such a plate we may make reference to the centeral images for faint comparison stars, and for a bright star under investigation we may take the mean of two symmetric dif-

fraction images.

The third method (the rotating sector) is used most often at the present time, but sometimes it is combined also with some of the other methods expounded here (for very bright stars); for instance we were enabled to compare the photographic image of Sirius with stars of the 10th magnitude i.e. as if its brightness was lessened by 11 1/2 magnitudes.

It is necessary to state here that of late American Observatories have been engaged in determinations of stellar parallaxes to a large extent then observatories elsewhere, namely, those of Allegheny, Dearborn, McCormick, Sproul (Swarthmore), Yerkes, Van Vleck and Mount Wilson; the Greenwich Observatory and the observatoty of Johannesburg in Sough Africa have also done much work in this field. All these observatories use large instruments of long focus and not only special photographic refractors, but also visual refractors; in the latter case, in the photographihg yellow filters of light and orthochromatic plates are used which help to exclude the influence of atmospheric dispersion although they lengthen the exposure. In the case of a long visual refractor "a leading eye glass" for the purpose of "holding on" to the star. This eye glass intercepts a part of the side rays in the tube. It is connected with the plate holder which may be slingtly shifted in two mutually perpendicular directions by special micrometric screws (Ritchy's plate holder).

The application of instruments of long focus (F = 7m to F = 20m) is advantageous in the sense of increasing the scale of the photographs which fact also increases the accuracy of the measurements even if this increase in accuracy is not proportional to the scale; but it lessens the field of view. We must especially note the Leander McCormick Observatory, Virginia, whcih has published more than a thousand parallaxes with a probable mean error of about +0".0094; also the Mount Wilson Observatory, California where the work is done with the aid of the largest reflector in the world (mirrors of 60 and 100 inches in diameter) enabling us to determine parallaxes even of very faint stars (the probable error of one parallax is about +0".007). We shall now expound those two different methods of treatment of plates for parallaxes which are used in the two American Obsdrvatories just mentioned.

c) The Application of the Stereocomparator (Blinkmicroscope and Stereomicroscope. At Mount Wilson A. Van-Maanen, a student and follower of Kapteyn, is applying the method of photographic determination of stellar parallaxes and proper motions. This method is essentially the same as that of Kapteyn except that no used is made of the latent image. The difference is that in this method the relative shifts of stars are measured (between the first and second or the third and fourth epochs, or, the distances P and Q) not on the same plate, but between a pair of plates placed in a Zeiss stereocomparator, as if optically

combined into one plate in the eye-piece of the blinkmicroscope or stereomicroscope. Usually an even number of plates is photographed (10 - 18), half of this number for the epoch close to the maximum positive, (and the other half for the maximum negative) influence of the parallax. Then these plates are connected in pairs in such a manner that the plate photographed near the meridian in the morning (the positive coefficient of the parallax with respect to the right ascension) is connected to the one photographed in the evening (negative coefficient). On the plates (6 - 10) faint comparison stars are selected around the star under investigation and a distance not greater than 8' from the star, and the rectangular coordinates of these stars are measured with respect to the star as the origin, with an accuracy of 0.1.

The plates are oriented in the comparator in such a way that the images of the same star on two plates must coincide optically, and the measures by the micrometer screw are made in the direction of the parallel of declination (along the X axis). It is evident that a very small portion of the screw (not more than 0.5 mm) is used and, of course, all precautions are taken for the exclusion of the personal error, of the possible influences of a change in temperature on the instruments, etc. Denoting by D the difference of the measures on two pairs of plates (in the sense: morning minus evening plate) as in Kapteyn's method, we may write the following for each measured star

$$a + b.x + c.y + \varphi_2 + \Delta p.\pi + \Delta m.\mu_\alpha = D,$$

where Δp and Δm are the differences of the coefficients of the parallax and of the proper motions between the pair of plates, and function: $a + b . x + c . y + \varphi_2$ are all the other influences on the distance (scale, orientation, refraction etc) and φ_2 is the sum of terms of the second order which may be neglected here, and the coefficients a, b, c, are constants for a given pair of plates. Proceeding in the same way as Kapteyn, Van-Mannen first assumes the parallaxes and proper motions of all comparison stars to be equal to zero; he determines a, b, c for each pair of plates by the method of least squares from equations of the form $a + b . x + c . y = D$, and the value of D for all stars is decreased by D_0 for the star the parallax of which is being determined (i.e. D is taken to be equal to zero). It is to be noted that the first terms of the conditional and normal equations remain the same for all pairs of plates.

Inserting the obtained values of a, b, c, in the equation given above we find the individual values of parallaxes of comparison stars with respect to the mean parallax from all those taken to be equal to zero. For the basic star, for which x = y = o, the following equation of conditional is obtained for each pair of plates;

$$\Delta p . \pi + \Delta m . \mu_\alpha + a = 0 \ (\text{For this } D = -a);$$

the solution of these equations of condition gives the final values of the parallax and of the proper motion of the star under consideration.

In 1912 the author of this article showed the possibility of the simultaneous determination of the annual parallax and of the proper motion stereoscopically - from the combination of three plates divided by intervals of half a year (see S. Kostinsky "On the Problem of Determination of Stellar Parallaxes Stereoscopically", Publications of the Academy of Sciences №11, 1912 in Russian).

d) Method of Measurement of the Rectangular Coordinates and Reductions of the Normal Plate.

Let us have a series of plates of the same region of the sky having the star under investigation nearly at the canter, and let us have the plates photographed at various epochs with respect to the influence of the parallax and the proper motion. If we select on the plate no less than three basic stars (the same for all plates) and then accurately measure on a Repsold instrument, or some other, the rectangular coordinates of all selected stars including the star under investigation, it is evident that we may compute strictly the equatorial coordinates of the latter for each plate separately by taking a certain system of equatorial coordinates for the basic stars. Then we may investigate the variations of these coordinates i.e. the change of the position of the star on the sky regardless of the causes of this change.

However, if we are not interested in the absolute (mean) position of a star, but only in its changes depending on the parallax or proper motion, it is sufficient to have only the approximate values of the absolute rectangular coordinates, but it is necessary to know accurately their differences between the individual plates. The computations may be greatly simplified if we reduce the measurements of all the separate plates to one Standard. Such a method was used by the older parallax investigators (for instance, by Bergstrand at Upsala), but the most convenient and the most widespread method at present is the one offered by Schlesinger in 1910 used mostly in American Observatories; it is called the "method of dependences" and it may be briefly stated as follows:

Let us denote by x_1, x_2, x_3, ... y_1, y_2, y_3, ... the rectangular coordinates, of comparison stars on some plate and by X_1, X_2, X_3, ... Y_1, Y_2, Y_3 ... the coordinates of the same stars on the normal or standard plate for which we may take any of the measured plates for fictitious mean one. We may write with a sufficient degree of apporximation (comp. Kapteyn's method and those of others) the following:

$$X_1 = x_1 + a.X_1 + b.Y_1 + c$$
$$X_2 = x_2 + a.X_2 + b.Y_2 + c$$
$$X_3 = x_3 + a.X_3 + b.Y_3 + c$$
$$\dots\dots\dots\dots\dots\dots$$

The number of such equations n is equal to the number of selected comparison stars. Presenting these equations in the form:

$$a.X_1 + b.Y_1 + c + (x_1 - X_1) = 0,$$

where the term $x_1 - X_1$ is known from accurate measurements, and solving this system by means of least squares we obtain the following normal equations (in Gauss's notation):

$$[X^2].a + [XY].b + [X].c + [X.(x-X)] = 0$$
$$[XY].a + [Y^2].b + [Y].c + [Y.(x-X)] = 0$$
$$[X].a + [Y].b + n.c + [(x-X)\quad] = 0.$$

From all the coordinates X_1 Y_1 on the normal or standard plate a constant number may be subtracted in such a manner that (X) = 0 and (Y) = 0; then the foregoing equations are simplified and they assume the following form:

$$[X^2].a + [X.Y].b = -[X.(x-X)]$$
$$[X.Y].a + [Y^2].b = -[Y.(x-X)]$$
$$+ \; n.c = -[x-X] = -[x],$$

whence we obtain:

$$a = \frac{[Y.(x-X)].[XY] - [X.(x-X)].[Y^2]}{[X^2].[Y^2] - [XY]^2} =$$
$$= 1 - \frac{[X.x].[Y^2] - [x.Y].[X.Y]}{[X^2].[Y^2] - [X.Y]^2}$$

$$b = \frac{[X.(x-X)].[XY] - [Y.(x-X)].[X^2]}{[X^2].[Y^2] - [X.Y]^2} =$$
$$= \frac{[X.x].[XY] - [x.Y].[X^2]}{[X^2].[Y^2] - [XY]^2}.$$

$$c = -\frac{[x]}{n}.$$

Denoting the measured coordinate of the basic star that is being determined on a plate by x_0, and its reduced value by X_0, we have:

$$X_0 = x_0 + a.X_0 + b.Y_0 + c = m.$$

Inserting here the previously obtained values of the coefficients a, b, c, we shall obtain a series of reduced quantities of the coordinate of the basic star: X_0, $(X_0)_1$, $(X_0)_2$, ... for all plates and they will differ among themselves only on account of the influence of the parallax and the proper motion of the star, and also on account of accidental errors of measurement. It remains only to obtain a solution for the system of conditional equations of the form:

$$P.\pi + t.\mu_x + Q = X_0 = m\ldots$$

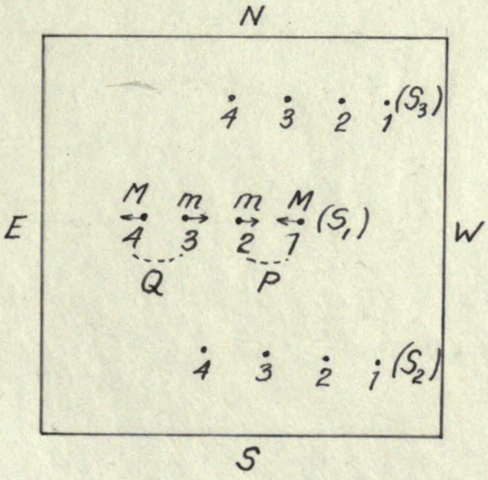

Fig. 17.

(the number of equations is equal to the number of plates) where P and t are coefficients of the parallax and the proper motion (along the coordinate X). In practice the values a, b, c are not computed separately, but in sorting their analytical values given previously into the formula for m, we obtain the latter as follows:

where the coefficients D_1, D_2, D_3 .... are functions of the coordinates of the comparison stars and of the basic stars and they are constants for all plates. These coefficients are called "dependences", and function m is called the "solution" of a plate.

Comparing these two methods of measurement and the reduction of the individual plates for the determination of parallaxes, we must say that Van Maanen's method has the advantage in the sense of a better exclusion of errors of measurement and besides it gives us an idea of the parallaxes of the comparison stars (which fast is important for the investigation of systematic errors). On the other hand, a more complicated apparatus (the stereocomparator) is used in it, and hence is subject more to the influences of temperature. In Schlesinger's method a simpler apparatus is used on which temperature has little influence, and the computations are simpler, but the measurements require more time and they are, so to speak, less "differential".

Of late, there has also been used for such measurements a method of comparison separate plates with a normal plate obtained through a glass and upon which are consecutively placed, layer upon layer, all other plates. Sometimes, instead of a normal plate, a plate of mirror glass with lines drawn on it is used (Greenwich, Upsala). In such a case the measurements are made through the glass which may be the cause of new errors requiring investigation.

e) Systematic Errors in the Determinations of Stellar Parallaxes. It was previously said that because of the extreme smallness of stellar parallaxes it is very difficult to ascertain the true magnitude of a parallax and to free it from remaining systematic errors depending on the kind of instrument used, on the type of measuring apparatus, on the method of measurement, on the degree of precautions taken during the work, on the personal equation the observer, etc.

Such errors may be of constant character for a given observatory under given conditions of work, i.e. they may increase or decrease all parallaxes by a certain constant magnitude. But there are also cases in which a series of parallaxes of various stars show a veriable systematic error as a function of the brightness of stars or in relation to their right ascension. These circumstances show the insufficient exclusion of the temperature.

That in the determination of parallaxes systematic errors of the types just indicated obtain more or less at all observatories has been definitely established. For instance, it is known that the Pulkovo normal astrograph yield parallaxes which are too large by 0".02 or 0".03; likewise the parallaxes

obtained at the Dearborn Observatory are too large by 0".03 and the degree of error differs depending on the difference of brightness between the star under investigation and the comparison stars; at the Sproul Observatory and at some other American Observatories small systematic errors (up to + 0".01) have been detected as a function of the right ascension of stars, etc.

Now, how are we to determine the systematic corrections for various series of parallax determinations as we know only the "probable" and not the truly accurate values of the parallaxes? Two methods are applied here: one of them is based on the statistical investigations of Kapteyn and his followers in regard to the mean (absolute) parallax of stars of different brightness, of various proper motions and spectral classes (see Publications of the Astronomical Laboratory at Groningen, Nos. 8 and 34, and also the separate dissertation by Van Rahijn, 1915). As a result of such investigations of trigonometric parallaxes, on proper motions, etc, tables are given which enable us to judge of the systematic correction of this or that series of determinations of parallaxes. In this manner Van Maanen determined in 1920 the corrections of 20 different series (according to observatories and persons) of parallaxes containing 1538 separate determinations for 901 individual stars.

Another more conditional method consists in a direct comparison of parallaxes of the same stars obtained in various conditions. In order to establish a certain normal system of parallaxes, it is necessary to assume certain hypotheses; for instance, to assume (as Schnauder does) that the sum of systematic errors for several observatories is equal to zero, or to reduce all parallaxes to the system of a single observatory (McCormick, for instance) which has most of the data (as was done by W. J. Luyten). The following table shows the systematic corrections determined by various methods at 5 observatories:

Table 2

Observatories	According to Van Maanen	According to Schnouder	According to Luyten
Allegheny	+ 0".004	+ 0".055	+ 0".002
Greenwich	+ 0 .012	--	- 0 .001
McCormick	- 0 .002	- 0 .000	- 0 .000
Mount Wilson	- 0 .002	- 0 .010	- 0 .006
Yorkes	+ 0 .004	+ 0 .003	- 0 .001

It appears that these corrections are, in general, small and not sufficiently reliable because of lack of sufficient data.

In conclusion of this section we shall say a few words as to which star ought to be selected for the investigation of their parallaxes. In earlier times the "parallax hunters" went after the largest parallaxes and hence preferred to investigate such stars whose proximity to the solar system appeared probable by various indications (greater brightness, larger proper motion, later spectral type, etc). In mass work and from the point of view of stellar statistics such a selection is not considered a desirable one; on the contrary the ideal thing would be the determination of parallaxes of as large a number of stars as possible for which all the characteristics previously mentioned would vary within wide limits. Unfortunately, such a work must be limited by necessity and, perhaps, the best limiting factor is the stellar magnitude; for instance: to pose the problem of determination of parallaxes of all stars up to the 7th magnitude inclusive; but even then the work would be tremendous: more than 20,000 parallaxes would have to be determined. (At the Leander McCormick Observatory the program is to include all accessible stars up to 4.5 magnitude inclusive).

On the other hand, it is of the utmost importance that the programs of observatories on the determination of parallaxes should overlap as much as possible as one of the best methods to lessen systematic errors is to transform them into casual errors by means of changing the conditions of observations measurements, etc. It would have enabled us to develop a genaral system of parallaxes.

At present the parallaxes of about 2000 stars are more or less well known. Special catalogues of parallaxes exist in which other stellar characteristics are also given: the approximate position of the star, its magnitude, proper motion and spectral class. Of these catalogues the best known are: Kapteyn's (1910), Schesinger(s (1924), and Cecchini's (1931).

§6. The Determination of Proper Motions of Stars. We know from astrophysical and other observations that all stars as well as our Sun move in space with various velocities, and in the long run, during centuries, we may assume this motion to be rectilinear and uniform. The projections of this motion on the celestial sphere in angular measure we call the "apparent proper motion" which needs be taken into consideration if we wish to have an accurate position of a star at an epoch far away from the epoch of the first determination of the position of the star. Since our observations are made from a point moving with the solar system, it is evident that the apparent proper motion results geometrically from the projection on the celestial sphere of the true individual motion of the star in space "peculiar motion" and of the projection of motion of the solar system "parallactic motion". We shall speak in more detail about these motions in Chapter III of this course.

The astrometric problem of determination of the apparent proper motion of stars is by far a simpler problem then the determination of parallaxes. Indeed, in the first place, we deal with greater quantities which can be "grown", so to speak, due to their secular character, and, in the second place, it is easier here to avoid many of the systematic errors of which we speak in the previous section because it is possible to repeat the observations under the

some conditions. The largest known annual proper motion at present is $10''.27$ of the star Proxima Ophiuchi (9.4) which is the nearest star to the solar system on the northern hemisphere (see 5). On the average the annual proper motion of all stars exceeds a parallax about ten or eleven times (according to Newcomb . In general, the problem is reduced to a comparison of two more positions of a star on the sky observed at epochs as far apart as possible. The problem, as a whole, may be subdivided into the following two problems:

1) The determination of the absolute proper motion of individual stars (or a group of stars) located on the entire sky or a large zone of the sky. The term "absolute" must be understood here conditionally, since in the determination of the direction of the proper motion or the motion in equatorial coordinates we must take into consideration the phenomena of precession and nutation.

2) The determination with respect to the proper motion of one of several stars on a small limited portion of the sky; for instance, the investigation of the proper motions of a group of stars up to a definite magnitude with respect to fainter stars in clusters, selected areas, etc.

The first problem is simply solved for the brighter stars since we have a series of visual or photographic stellar catalogues composed of observations at various epochs. Selecting from them the positions of the stars under consideration, the positions are reduced to one normal equinox, and the selection of the "precession constant" determines to a certain extent the system of the obtained proper motions. Next, it is of the utmost importance that the positions of a star (α and δ) taken from each catalogue be reduced to a certain definite system which is usually the system of the so called fundamental catalogues of Auwers or of Boss. For most visual catalogues such reductions to the fundamental system are known from special investigations.

Let us have for a series of epochs $t_1, t_2, t_3, \dots$ expressed in fractions of a Julian year a corresponding series of values of equatorial coordinates of a certain star $\alpha_1, \alpha_2, \alpha_3, \dots \delta_1, \delta_2, \delta_3, \dots$ reduced as we had previously indicated. Denoting

$$\frac{\Sigma t}{n} = t^\circ, \qquad \frac{\Sigma \alpha}{n} = \alpha_o \quad \text{and} \quad \frac{\Sigma \delta}{n} = \delta_o ,$$

where n is equal to the number of catalogues used, we take the quantities α_0, δ_0 as the most probable values of the coordinates for epoch t_0 and, subtracting them from the individual values for each catalogue and setting $t - t_0 = \Delta t$, $\alpha - \alpha_0 = \Delta\alpha$, $\delta - \delta_0 = \Delta\delta$, we find the probable annual proper motions of the star by the formulas

$$\mu_\alpha = \frac{[\Delta\alpha . \Delta t]}{[\Delta t^2]} \quad \text{and} \quad \mu_\delta = \frac{[\Delta\delta . \Delta t]}{[\Delta t^2]} .$$

(in the notation of Gauss).

If the positions of a star given by the catalogues are of various accuracies, the conception of the weight of each catalogue is introduced and the values of the weights are introduced into the formulae in the usual manner.

The methods of solution of the second problem - the determination of the relative proper motions over a small area (for instance, over the area of a plate of a normal astrograph) were essentially expounded in 5, as the photographic method of observations is almost exclusively used at present, and the difference form the method of the determination of the annual parallaxes consists merely in the following:

1) The photographs under comparison should be removed in time from each other as far as possible (10 - 20 years) and be photographed by the same instrument:

2) the photographs should be made at the same hour angles and under the same conditions as much as possible (sensitivity of plates, exposure, condition of atmosphere, etc). The photographs may also be made on a single plate by Kapteyn's method of using the latent image; then they are developed as we have already stated (see 5, equation 18). The difference is that the plate is measured in both coordinates and not merely in X. By this method Kapteyn determined the proper motions of several thousands stars, with a probable error of only ± .011, using plates obtained at Helsingfors at intervals of 8 years. However, it appeared difficult to preserve the underdeveloped plates for 8 - 9 years since the latent image of a star gradually disintegrates and may become just a gray spot the measurements of which would be inaccurate. Therefore, this method is now abandoned and photographs are taken at large intervals of time on separate plates.

The measurements of plates for the determination of proper motions are made exactly in the same way as described in § 5 for parallaxes, namely:

1) by the method of optical connection of two plates on a stereocomparator (blinkmicroscope or stereomicroscope) and measuring by the micrometer of the small shifts of two images;

2) by measuring the accurate rectangular coordinates x and y on the universal instrument separately for the two plates and then imagine that we place them one upon the other, i.e. using only the accurate residuals Δx and Δy fpr further reduction;

3) by photographing the plate at the second
placing them together on the universal instrument face to face so that only the small distances are measured by the micrometer.

In all these cases the computation of the obtained measurements are made by the same formula of Kapteyn the following form:

where μ is the annual proper motion, p is the measured shift in the corresponding coordinate between the epochs t_1 and t_2, x and y are the approximately known rectangular coordinates of the star, and A, B, C are constants for a given pair of plates determined by the method of least squares with respect to all the stars measured on the plates, or by means of certain basic stars or compariosn stars. Considerations similar to those described in 5 show that in such a case $\mu = 0$ is assumed for this group of stars, and hence, the proper motion of each star is determined with respect to the mean motion of the stars of that group. If this mean (absolute) motion in the case of bright basic stars is determined from meridian observations, then it is possible to reduce the relative proper motions of all stars to absolute ones. Sometimes such a reduction is done theoretically by the comparison of the obtained relative proper motions with statistically computed parallactic motion of the stars in the given region of the sky as a function of their photographic magnitude. In the same manner it is possible to investigate those remaining systematic errors in the proper motions which are of the charactér of the "light equation" (see, for instance: Van Rhijn, "The Proper Motions of the Stars in the Paresepe Cluster" Publications of the Astronomical Laboratory at Groningen, 26, 1916). In connection with this, and also in connection with the problems of stellar statistics in general, the problem of determination of absolute positions and proper motions of the faint stars is of great importance. The ways of solving it are indicated in 4 of this Chapter (photographing of AG zones), and it is quite probable that the application of very wide angled photo objectives and of the chain method in connection with similar observations of basic stars by transit instruments will be the best way in solving this problem. We shall add to this that some of the observatories, which have participated in the work on the international may of the sky, are now repeating the photographs of their zones and derive systematically the proper motion of numerous faint stars by the differential method (for instance, the Helsingfors and Greenwich Observatories).

Of the three methods, mentioned previously, of the photographic determinations of relative proper motions of stars the first method (by the stereocomparator) is used most often; then the second is used also a great deal; the third method (measurement through a glass) is still in the stage of investigation and improvement. The application of the first two methods at Pulkovo gave a probable error for each proper motion of only ± 0".006 for a difference of epoch of about 10-12 years; in individual cases of larger proper motions by the stereoscopic method the probable error was as low as ± 0".0046 for a difference of epochs of ten years. Numerous discoveries of noticeable proper motions were made by the first method at various observatories, especially by Wolf at Heiedlberg.

At the present time (in 1931) the proper motions of about 42000 stars, chiefly up to the 10th magnitude, are known from photographic and meridian observations, but the larger motions of much fainter stars are also known. There are a few special catalogues of proper motions. Of these catalogues we shall mention only the two following ones:

1) R. Scharr, Eignbewegungs-Lexikon, 1923 (with six additions up to 1931

2) Catalogue of Proper Motion Stars (Pulications of the Cincinnatti Observatory, 1930 (contains only large proper motions > 0".40 for 1475 stars).

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